THERMAL & FLUIDS ANALYSIS WORKSHOP

GSFC · 2015

Bayesian-based Simulation Model Validation for Spacecraft Thermal Systems

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Presentation Overview

- **Introduction**
  - Background and Motivation
  - Literature Review
  - Research Goal
- **Bayesian-based Model Validation (BMV) Methodology**
  - Methodology Overview
  - REXIS Solar X-ray Monitor (SXM) Case Study
- **Conclusion**
  - Primary Contributions
  - Recommendations for Future Work
  - Acknowledgements

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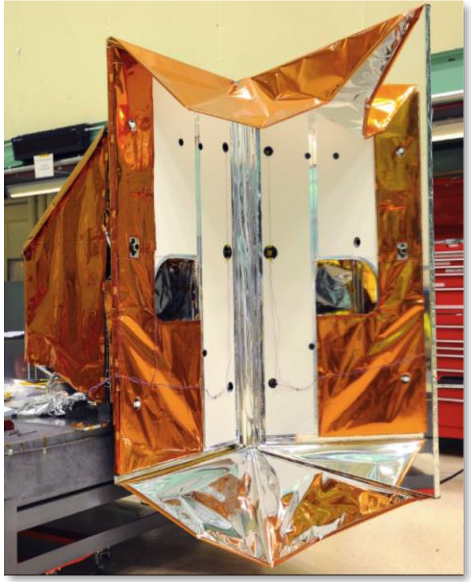
Motivation

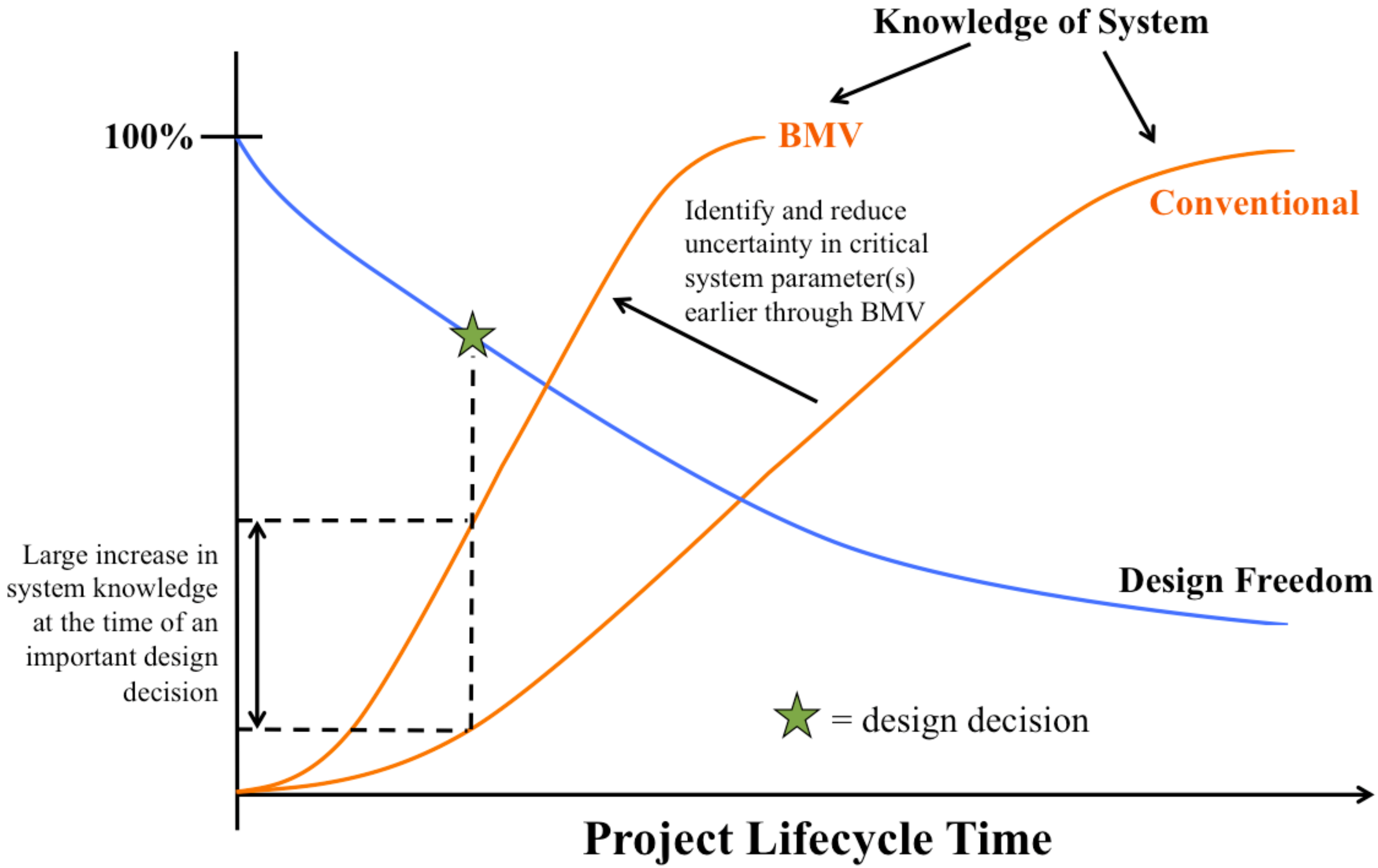
How effective are current model validation practices?

- Literature review of flight temperatures vs. model predictions
- Thermal systems are successful but:
  - Overdesign w.r.t. stacked worst case scenarios
  - Occasional model inaccuracies



Improve thermal model validation process to reduce *form*-related and *process*-related costs long term

Bayesian-based Model Validation (BMV) Motivation



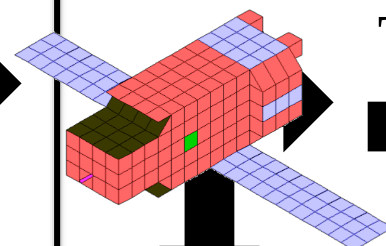
Potential to increase knowledge of the system earlier in the project lifecycle when important design decisions are made

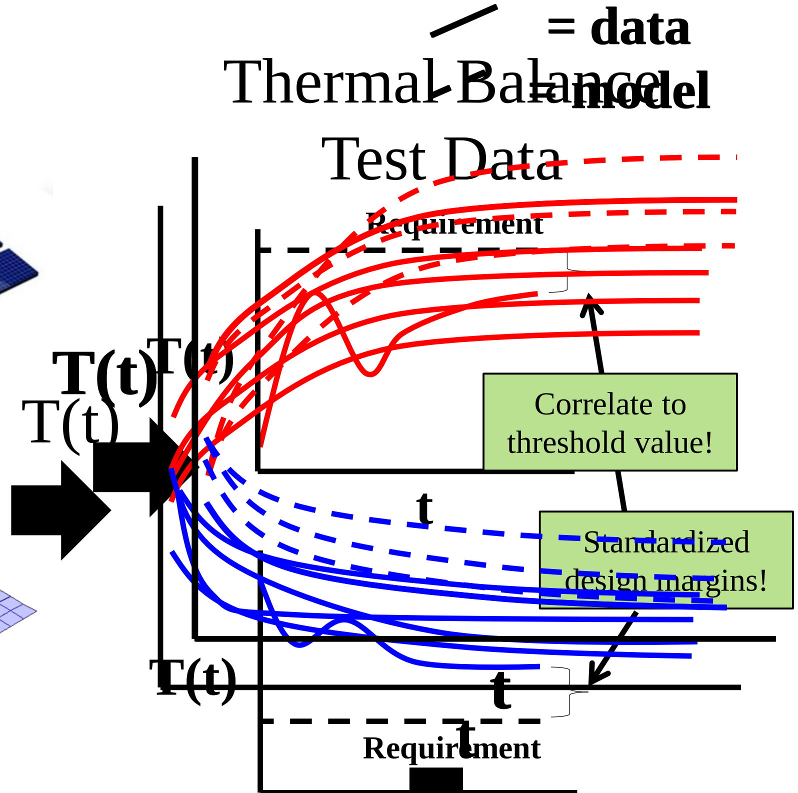
A Conventional Model Validation Approach

Thermal Vacuum Chamber

Model Parameter	Cold Case Value	Hot Case Value
x_1		
x_2		
x_3		
⋮		
x_N	4.4	1.2

Test/verification Standards





Literature Review Summary and Research Goal

Summary of Literature Review		
Area	**State of the Art**	**Thermal Convention**
Uncertainty Propagation (UP)	Probabilistic uncertainty characterization; UA/GSA [13,14]	Convex uncertainty characterization; margin "downstream" of model [15-17]
Design of Experiments (DOE)	Optimal (Bayesian) Experimental Design [20,21]	Classical DOE [15-19]
Model Calibration	Bayesian [24-27] (K-O approach [22,23])	Manual model correlation [1]

Research Gap

In practice, the state of the art methods are used rarely and in limited capacity

- No existing framework to combine state of the art methods for thermal systems

Research Goal

Improve the thermal model validation process by developing a tailored methodology that combines the state of the art validation methods of Uncertainty Quantification (UQ) and Design of Experiments (DOE).

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  - Research Goal and Thesis Objectives
- **Bayesian-based Model Validation (BMV) Methodology**
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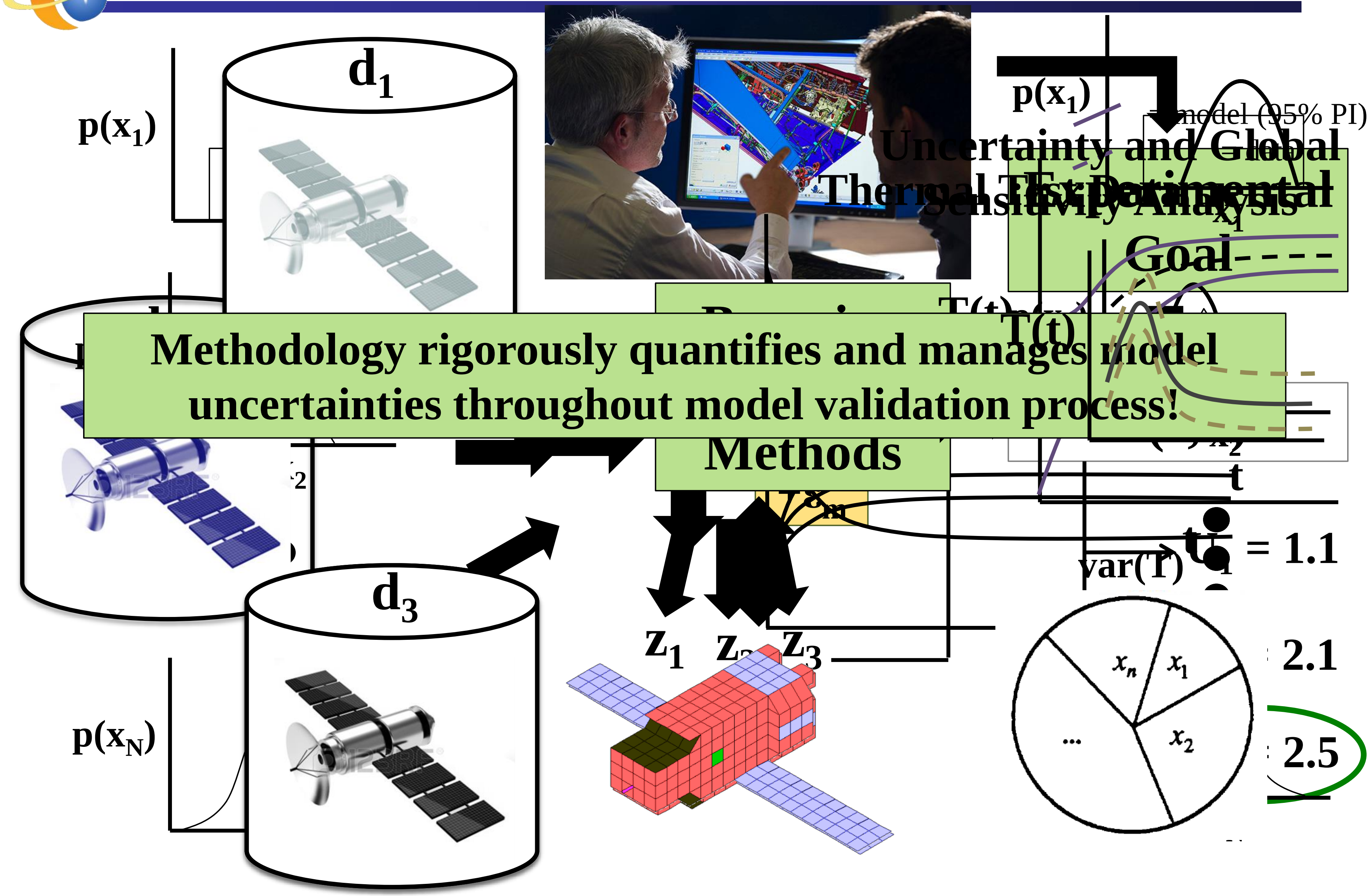
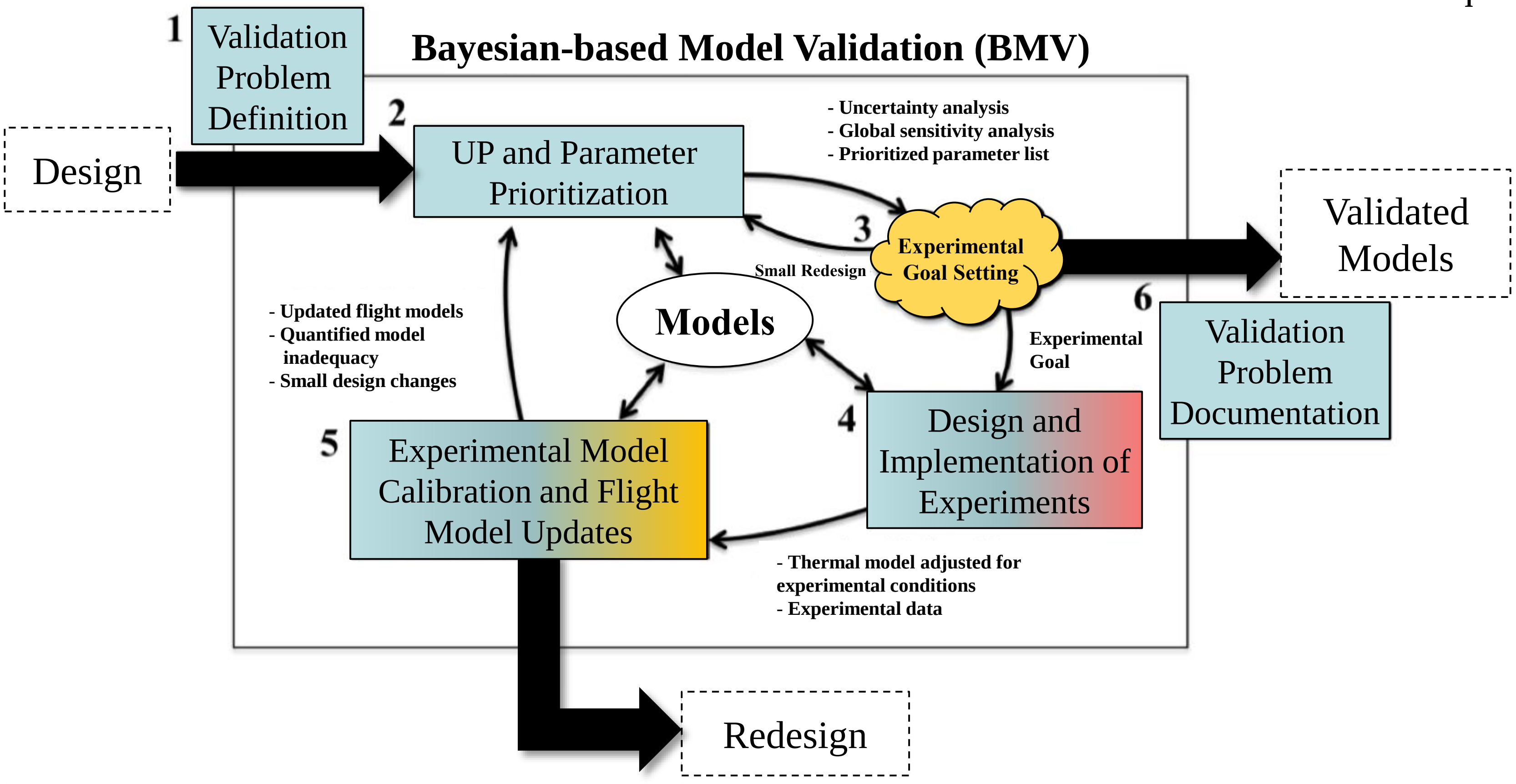
Visualization of Proposed Methodology



Methodology rigorously quantifies and manages model uncertainties throughout model validation process!

BMV Methodology Overview

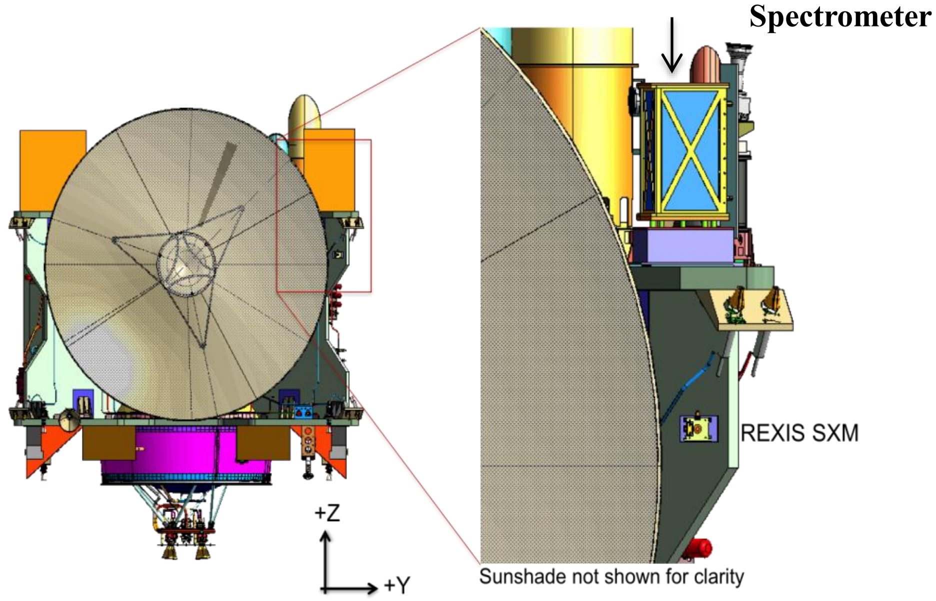
Blue = analyses
Orange = decision
Red = hardware required

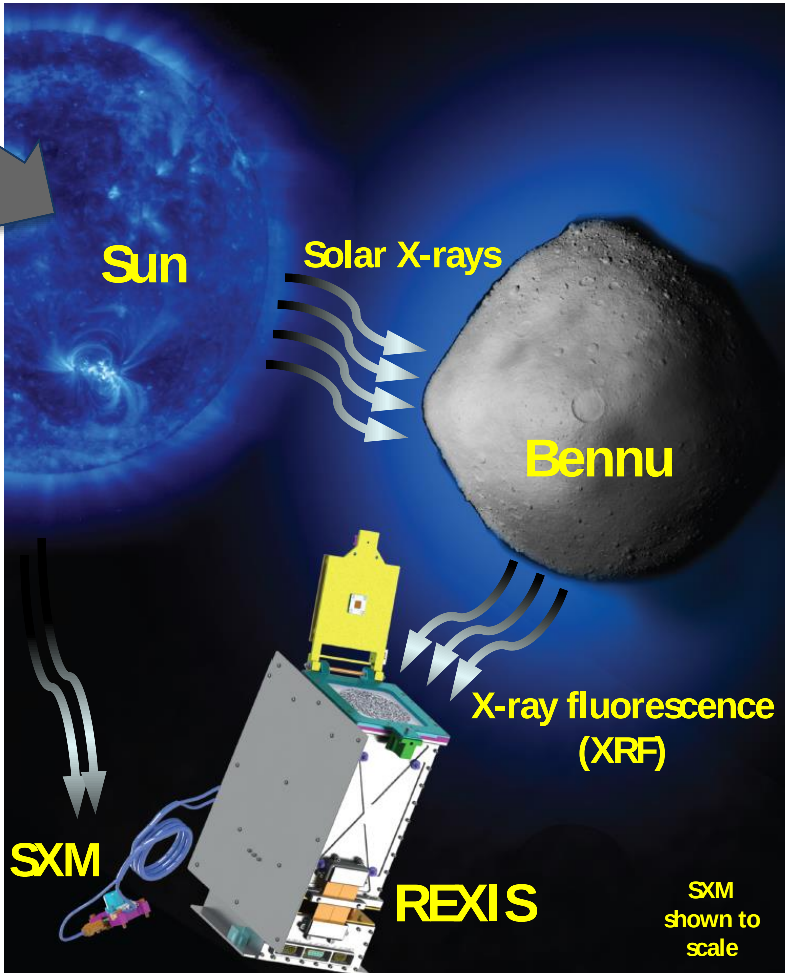


REXIS Solar X-ray Monitor (SXM) Case Study

REgolith X-ray Imaging Spectrometer (REXIS)

- One of five payload instruments on OSIRIS-REx
- Complements and enhances other science instruments on OSIRIS-REx
  - Characterizes Bennu among known meteorite groups and map surface elemental distribution

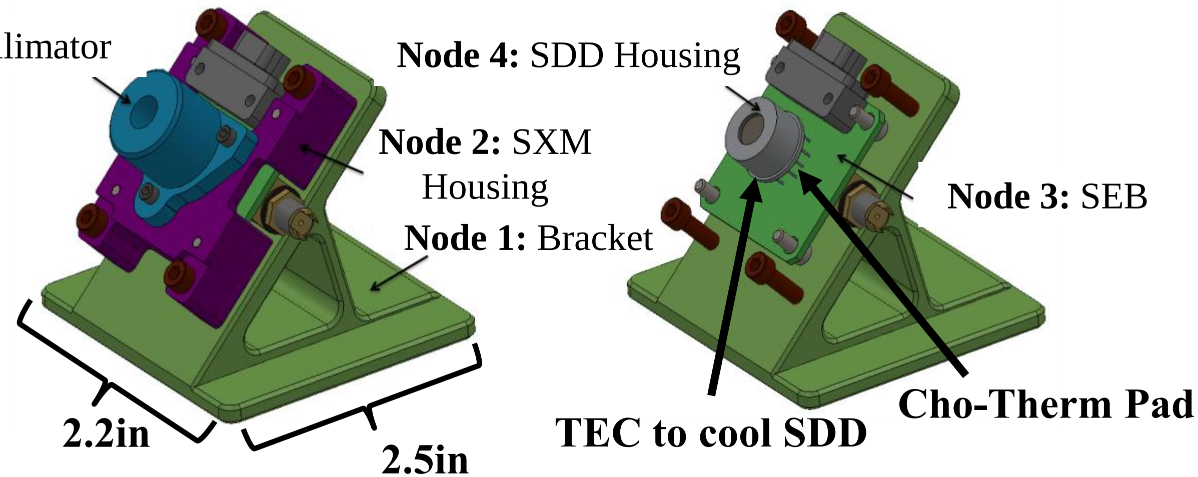




Two assemblies: spectrometer and Solar X-ray monitor (SXM). SXM observes time-variant solar X-ray spectrum to provide context to spectrometer measurements.

Thermal Model Structure

- Five node lumped parameter model
- 38 total parameters
- 18 uncertain parameters



Model Structure

Environment

Deep Space							
	Sun		~~R~~	~~R~~		R	R
		O-REx					
		C	1 – Bracket				
			C	2 – SXM Housing			
				C	3 – SEB		
				C	C	4 – SDD Housing	
R				C			5 – Collimator

SXM

$$\mathbf{y} = \eta_{SXM}(\mathbf{x})$$

Lumped parameter model provides *y(x,t)*, temperatures versus time for each node

What is max allowable $T_{\text{O-REx}}$?

SXM Thermal Requirements

At least 99% probability that all temperature ranges are satisfied

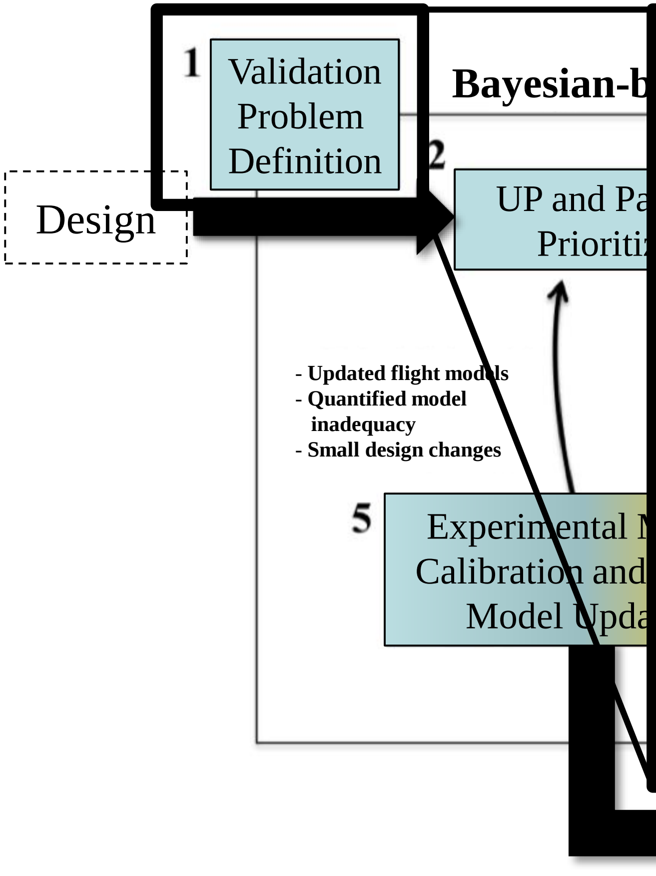
Component	Operational (^{o}C)	
	Min	Max
SDD Housing	-40	100
SEB	-40	85
SDD	-100	-30

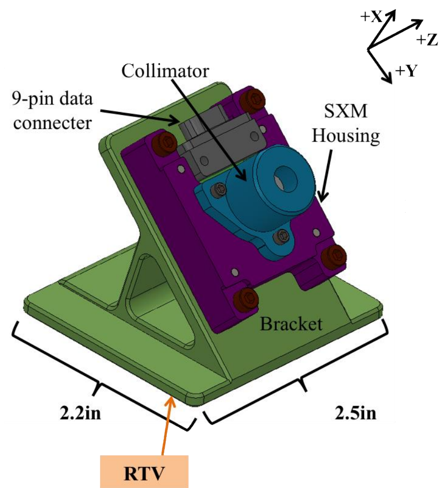
SDD = silicon drift detector
SEB = SXM electronics board

Three quantities of interest (QoIs) for SXM – all operational component temperature ranges

Summary of SXM Case Study

Blue = analyses
Orange = decision





Component	Operational (^{o}C)	
	Min	Max
SDD Housing	-40	100
SEB	-40	85
SDD	-100	-30

- Three QoIs
- What is max allowable $T_{O\text{-}REx}$?

Uncertainty Analysis

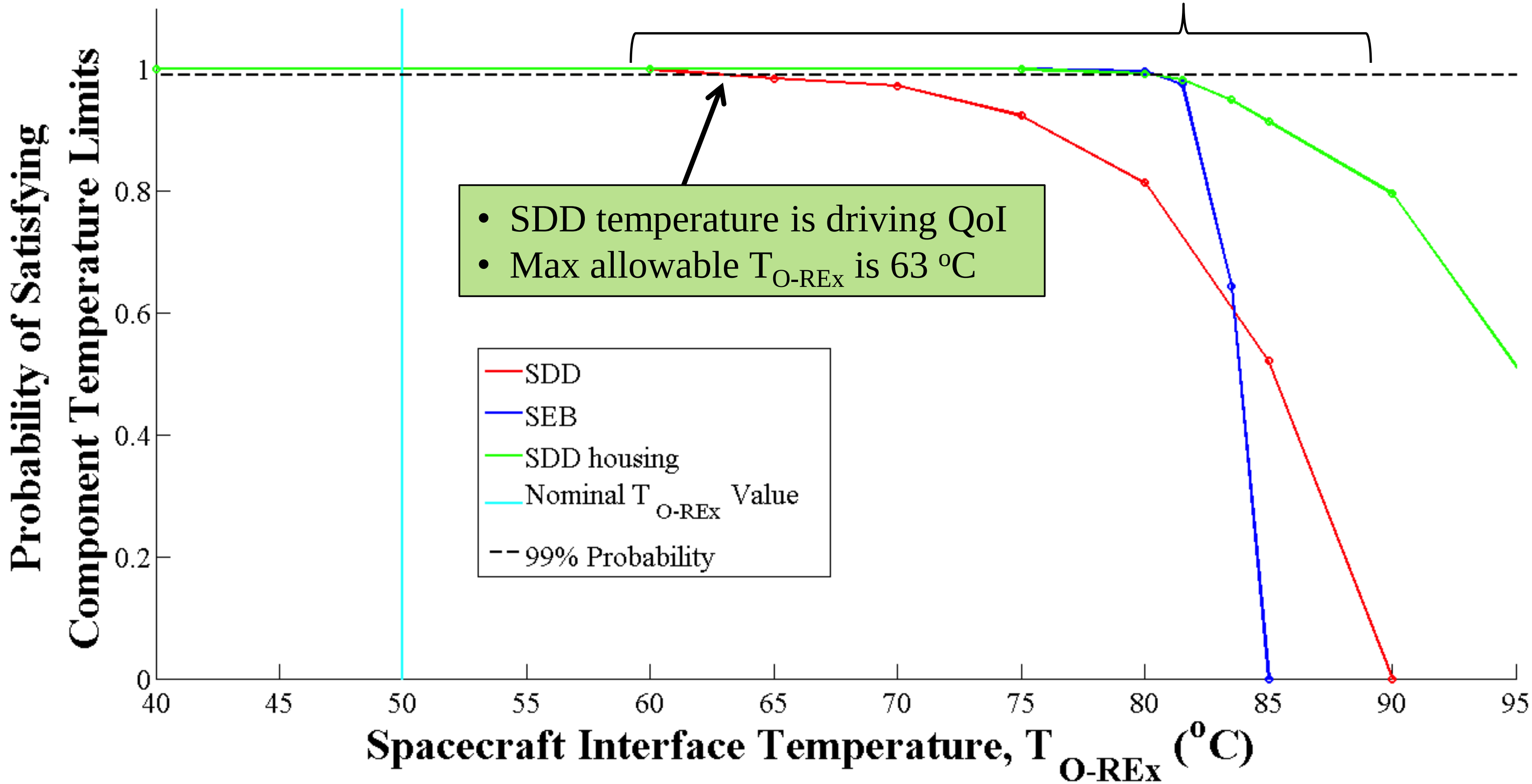
Monte Carlo (MC) Simulation

$$\overline{\eta}_{SXM,N} = \frac{1}{N}\sum_{i=1}^{N}\eta_{SXM}(\mathbf{x}_i)$$

x contains all system and environmental parameters

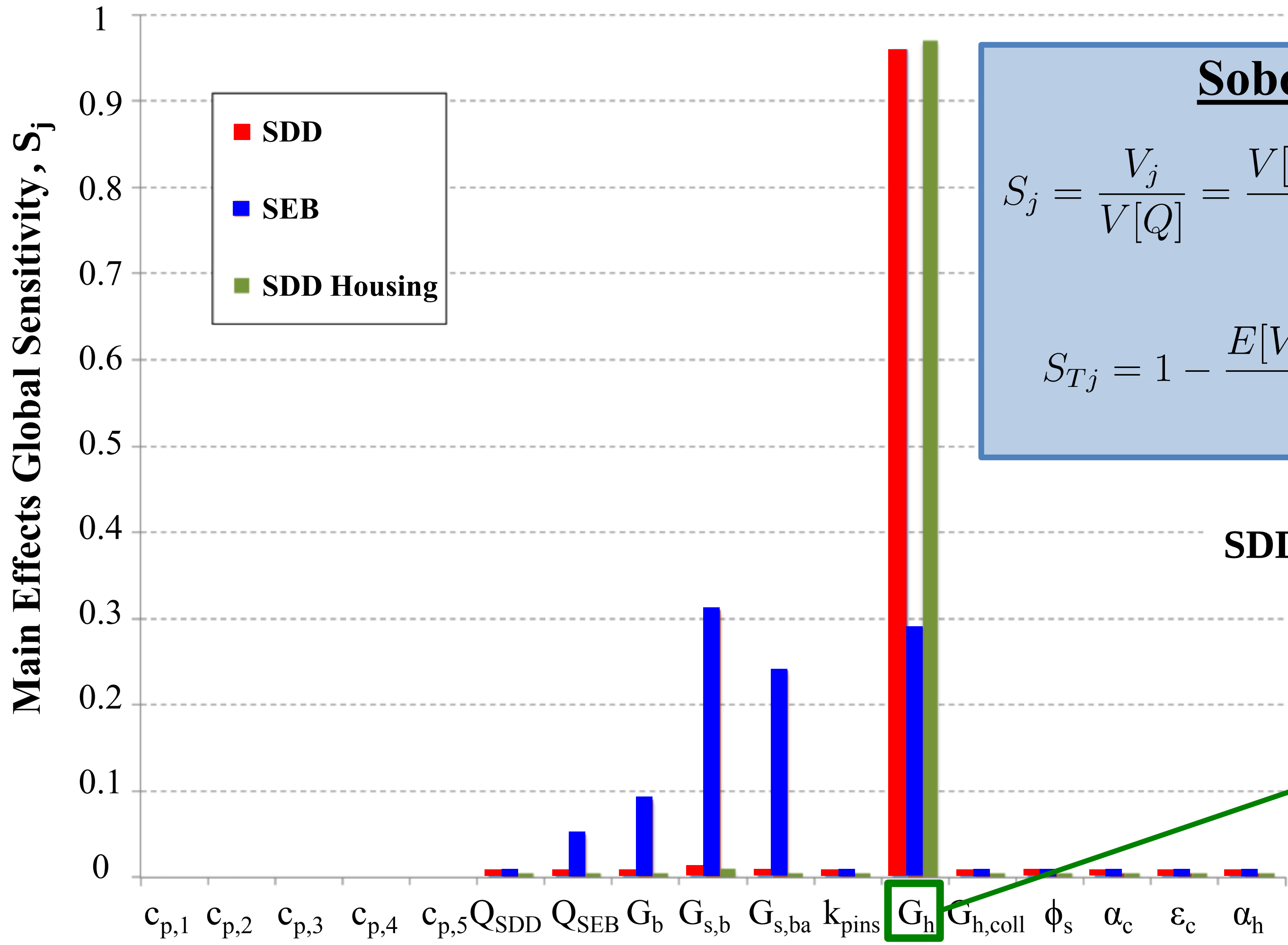
Parametric uncertainty results in uncertainty in max allowable $T_{O\text{-}REx}$

- SDD temperature is driving QoI
- Max allowable $T_{O\text{-}REx}$ is 63 °C



Global Sensitivity Analysis

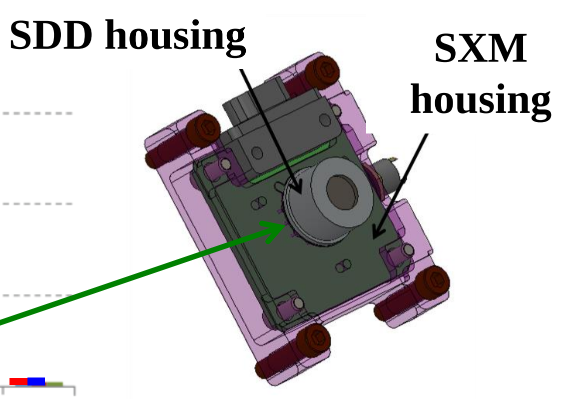
Main Effects Sensitivities for $T_{O\text{-}REx}$ = 85 ºC



Sobol' Indices

$$S_j = \frac{V_j}{V[Q]} = \frac{V[E[Q|x_j]]}{V[Q]} \quad \Big\} \text{ Main effects}$$

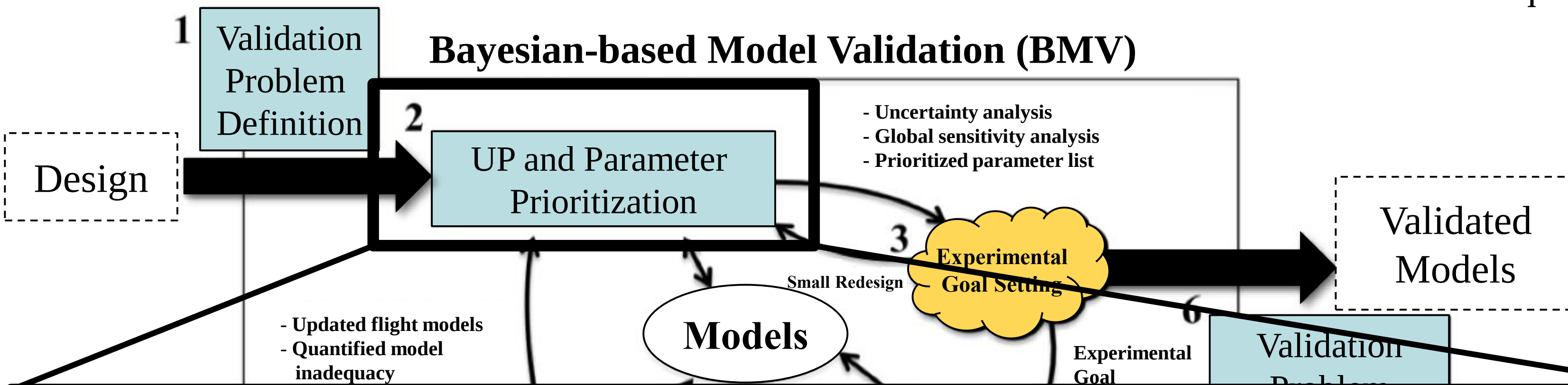
$$S_{Tj} = 1 - \frac{E[V[Q|\mathbf{x}_{\sim j}]]}{V[Q]} \quad \Big\} \text{ Total effects}$$

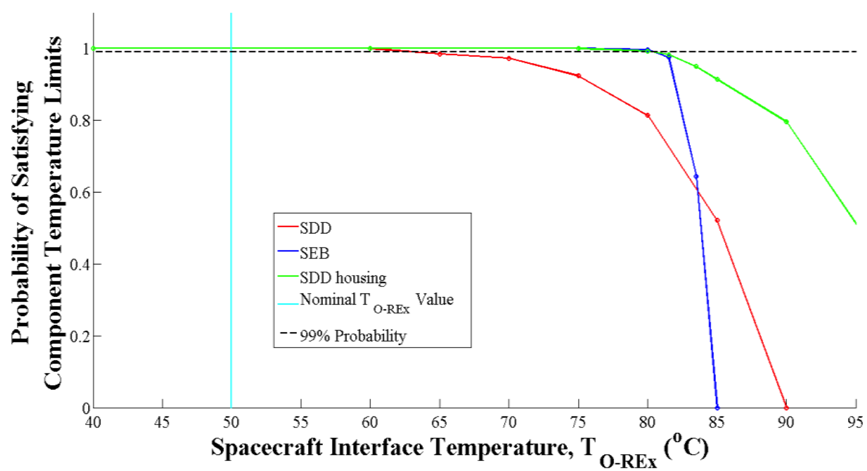


Conductance between SDD housing and SXM housing, G_h, is driving uncertain parameter for SDD and SDD housing temperatures

Summary of SXM Case Study

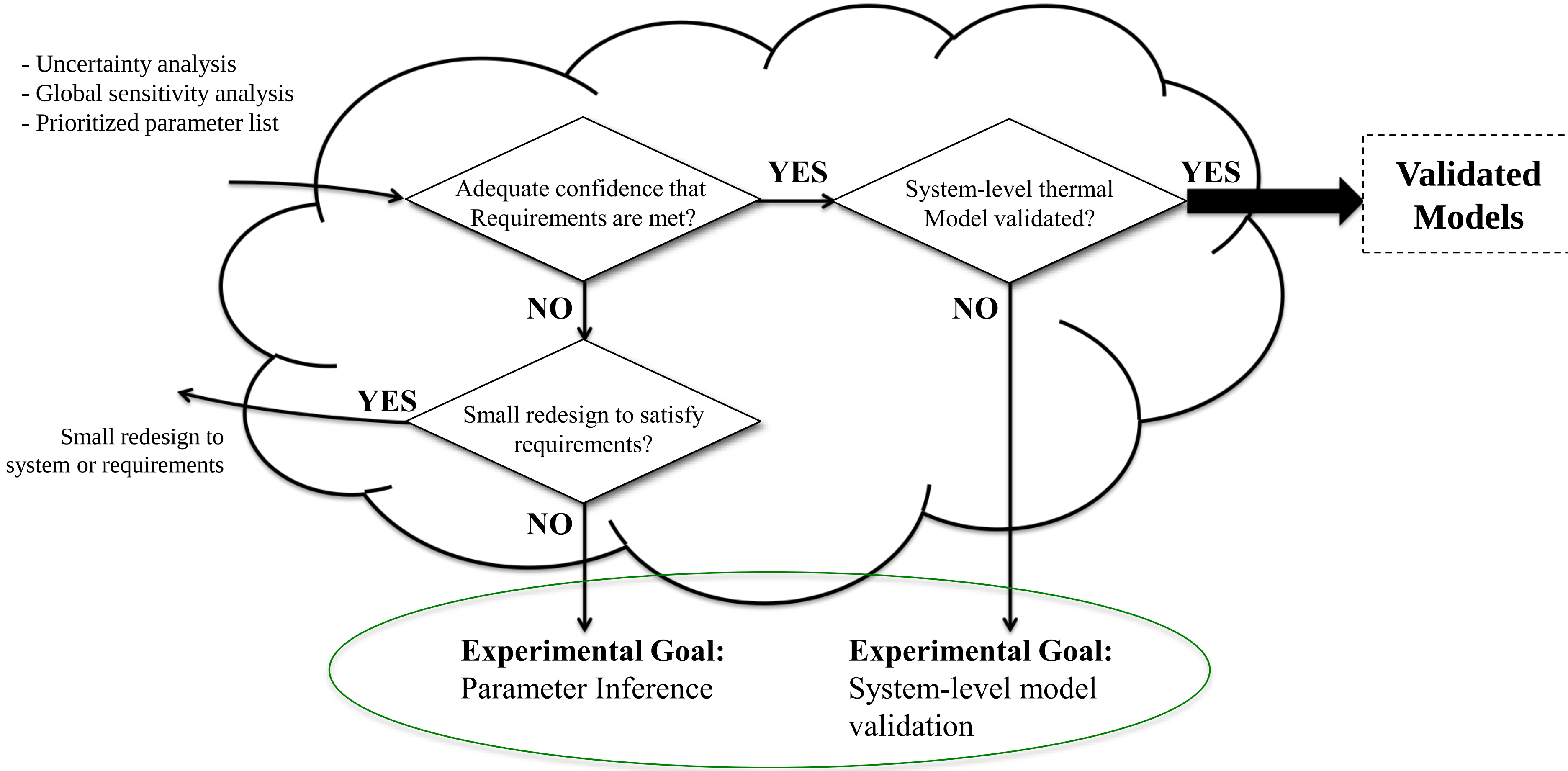
Blue = analyses
Orange = decision
Red = hardware required





- SDD temperature = driving QoI
- SDD temperature below 99% at $T_{O\text{-}REx}$ = 63 °C
- GSA indicates G_h parameter as the primary global sensitivity

Experimental Goal Setting



Two types of experiments will be implemented:

- Parameter inference experiment to reduce uncertainty in G_h
- Model validation experiment to validate SXM thermal model

Summary of SXM Case Study

Blue = analyses
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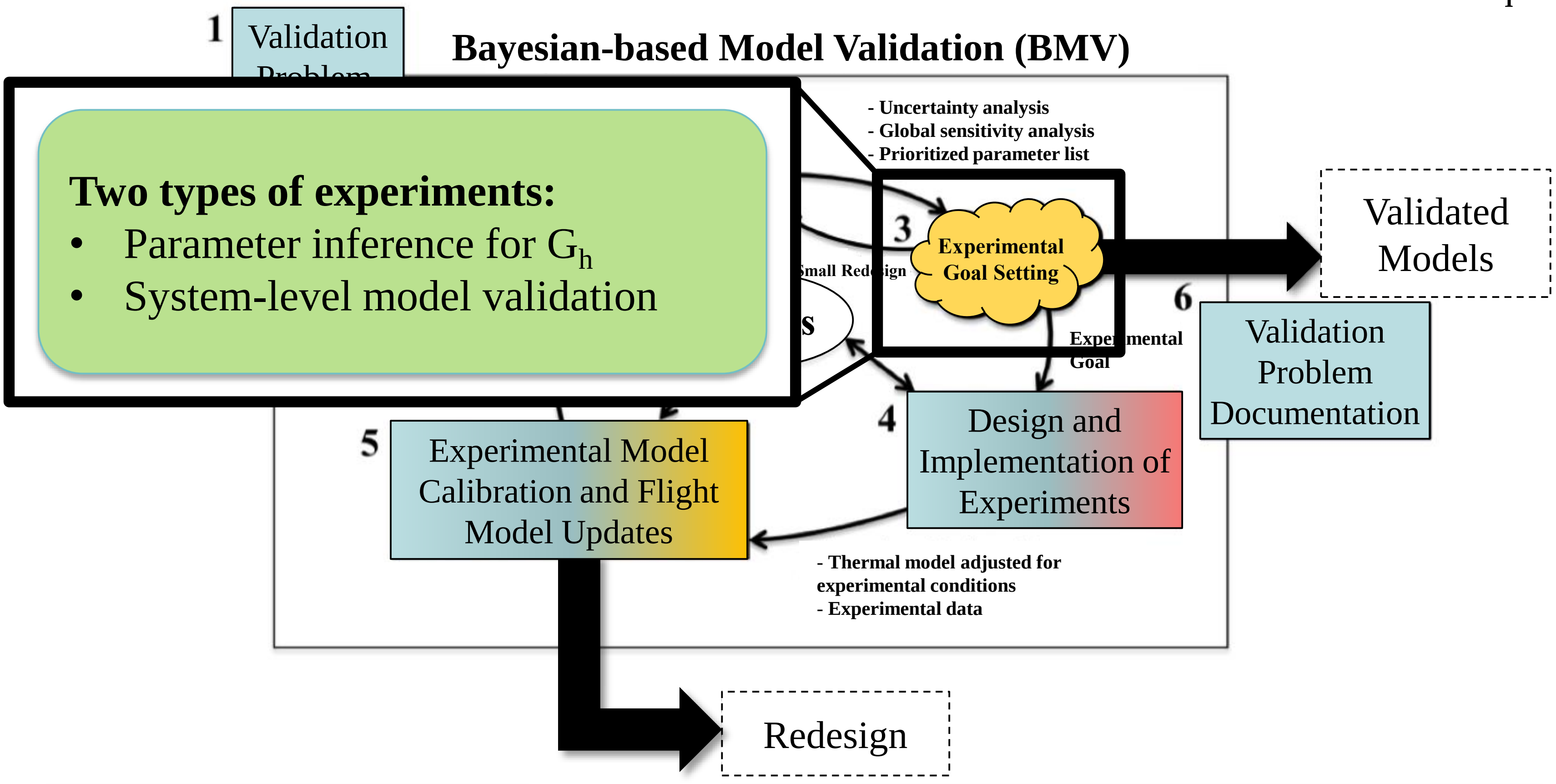
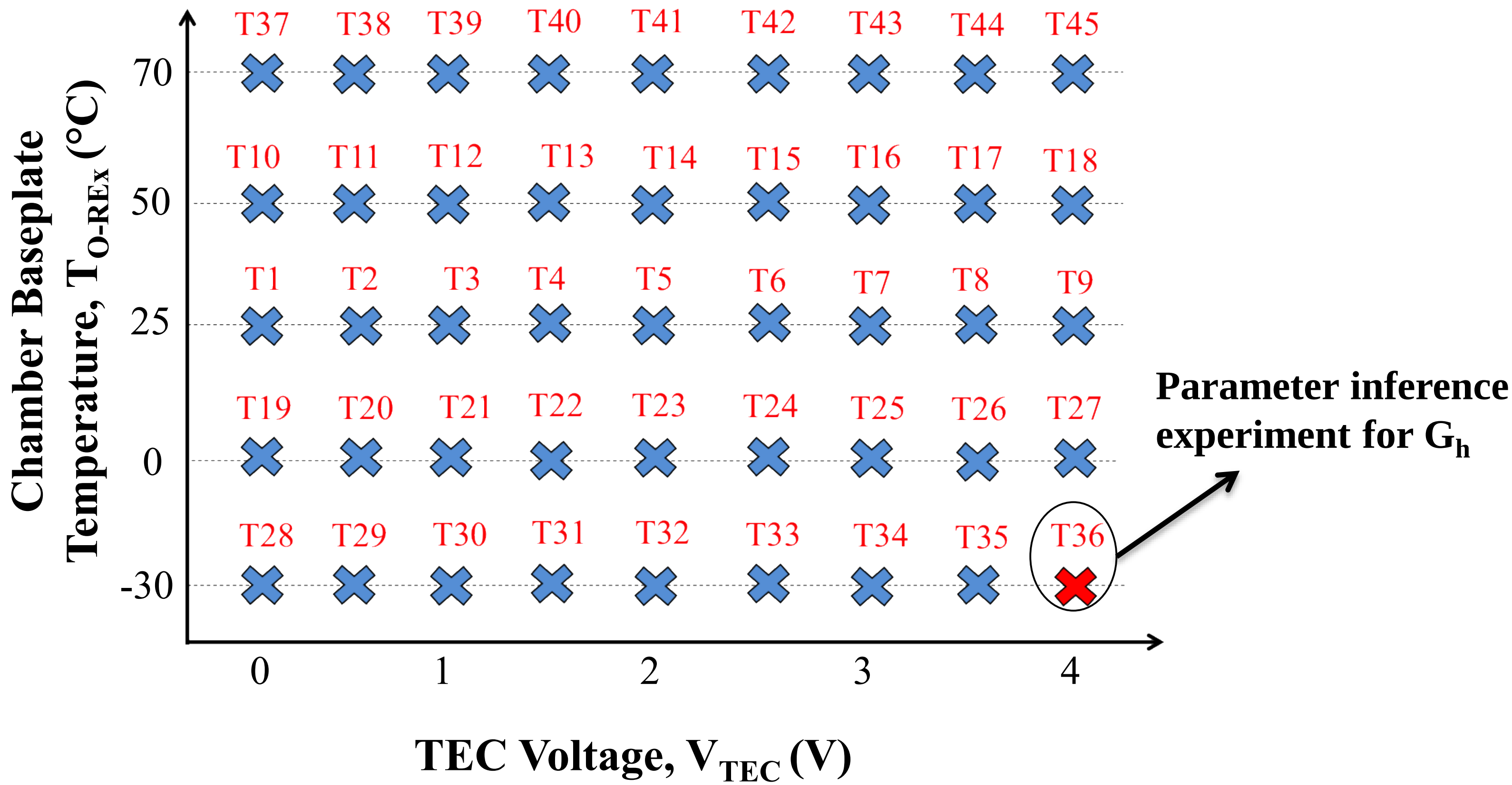
Red = hardware required



Model Validation Experiment



- Full-factorial experiment (classical DOE)
- Small system time constant
- All test phases completed to steady state conditions

Validation experiment designed to span domain of expected TEC voltages and SXM interface temperatures

Parameter Inference Experiment

Nomenclature

x: all model parameters

θ: parameter(s) of interest, $\theta = G_h$

d: experimental conditions,

d = $[T_{O\text{-}REx}, V_{TEC}]^T$

z: experimental result/data

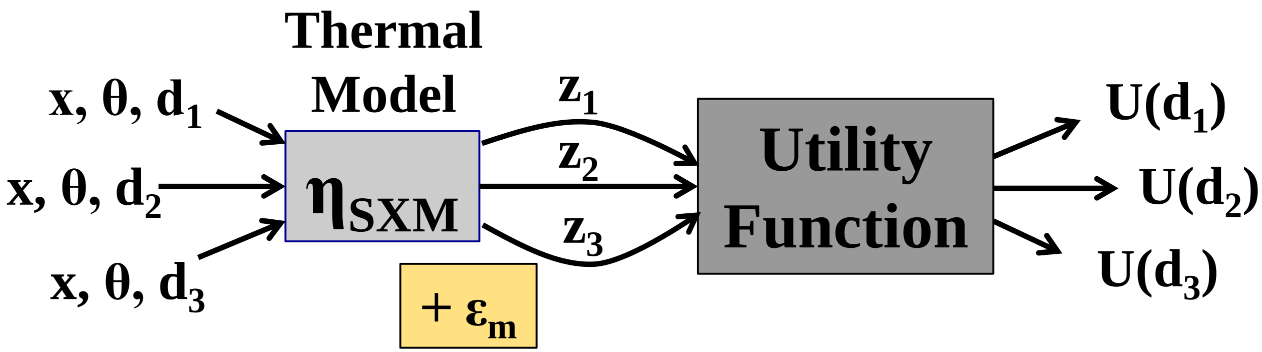
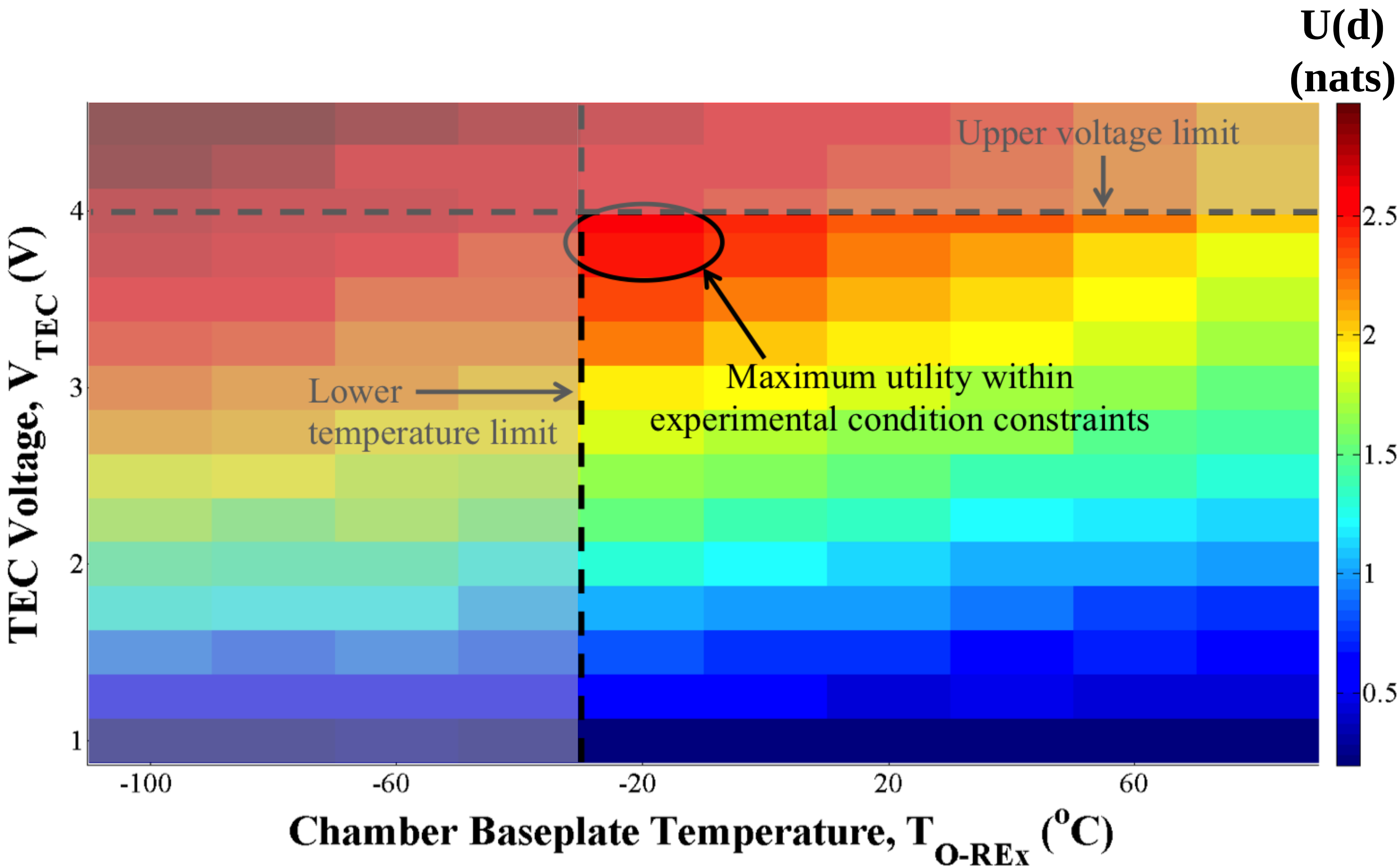


Table of Experimental Design Conditions, d

Name	Variable	DOE Variable	Units	Nominal Value	Minimum Value	Maximum Value
Temperature of O-REx Deck	$T_{O\text{-}REx}$	d_1	°C	40	-100	75
TEC Voltage	V_{TEC}	d_2	V_{DC}	3.0	0	4.5

Two experimental design conditions varied to create different parameter inference experiments for G_h

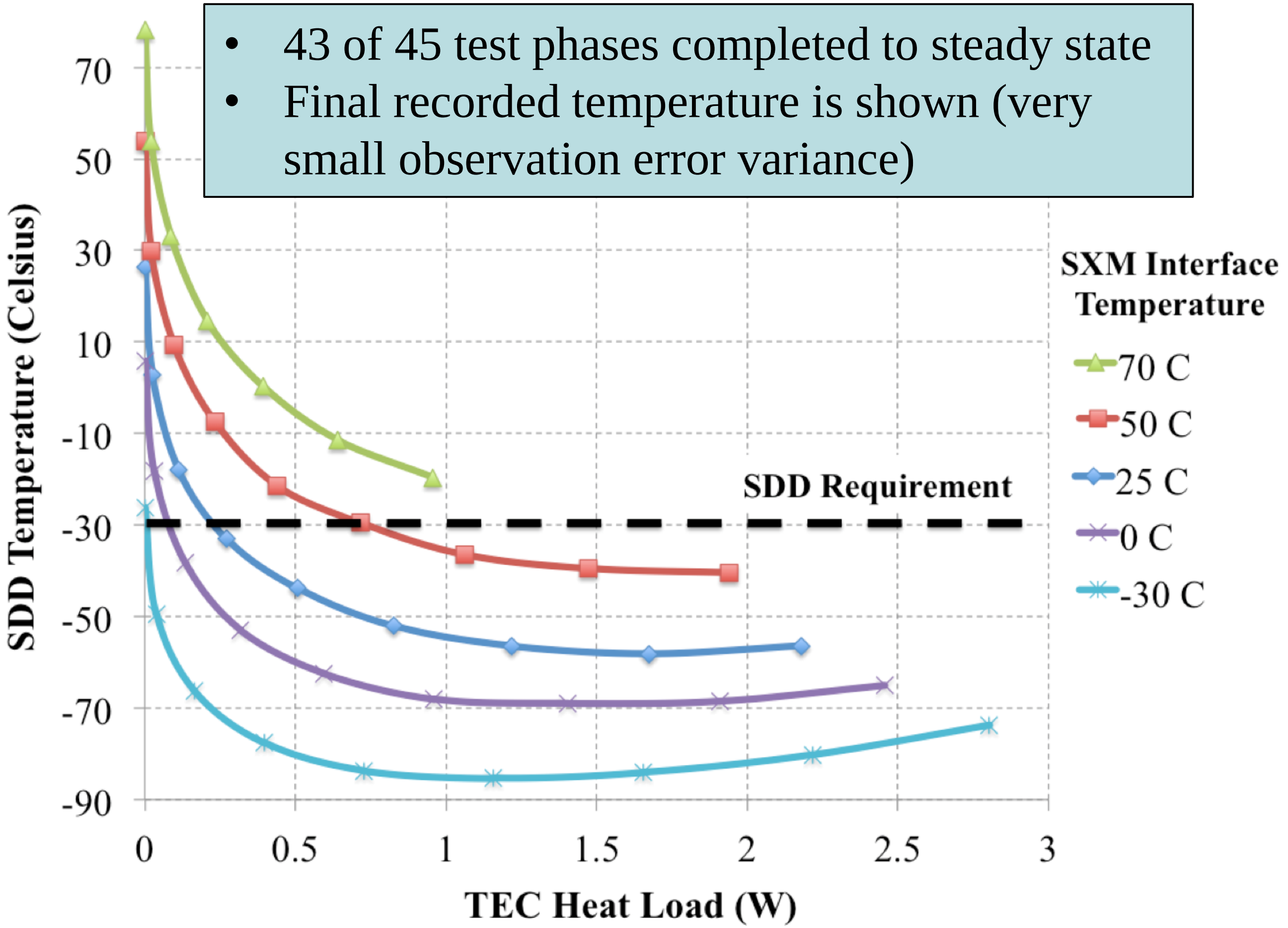
Parameter Inference Experiment



d* is $T_{O\text{-}REx}$ = -30 °C, V_{TEC} = 4.0 V, and T_w = 23 °C

Sample Experimental Results

- 43 of 45 test phases completed to steady state
- Final recorded temperature is shown (very small observation error variance)



- Max expected TEC power is <2.0 W
- Preliminary thermal model has good predictive accuracy for SDD temperature

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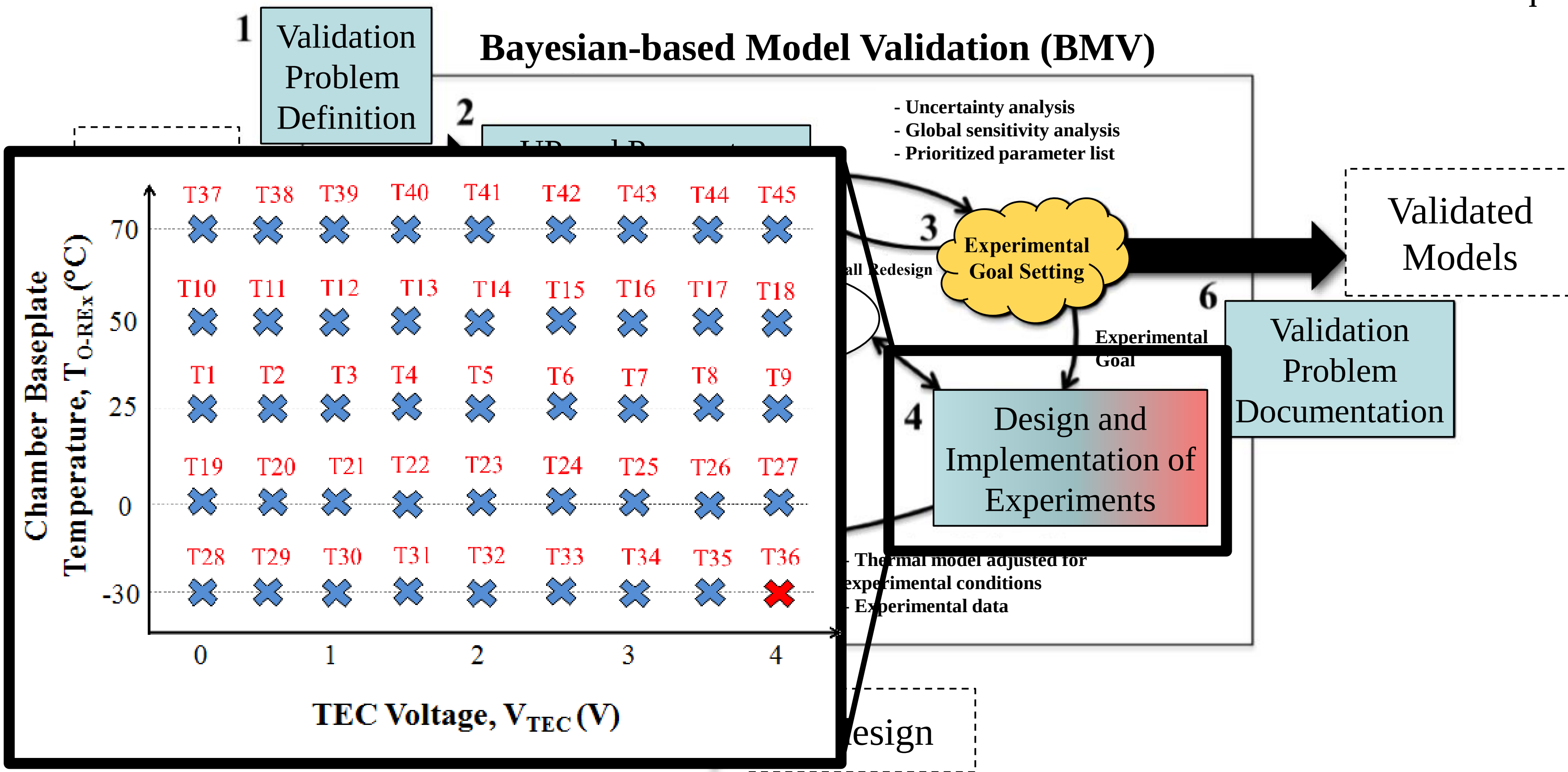
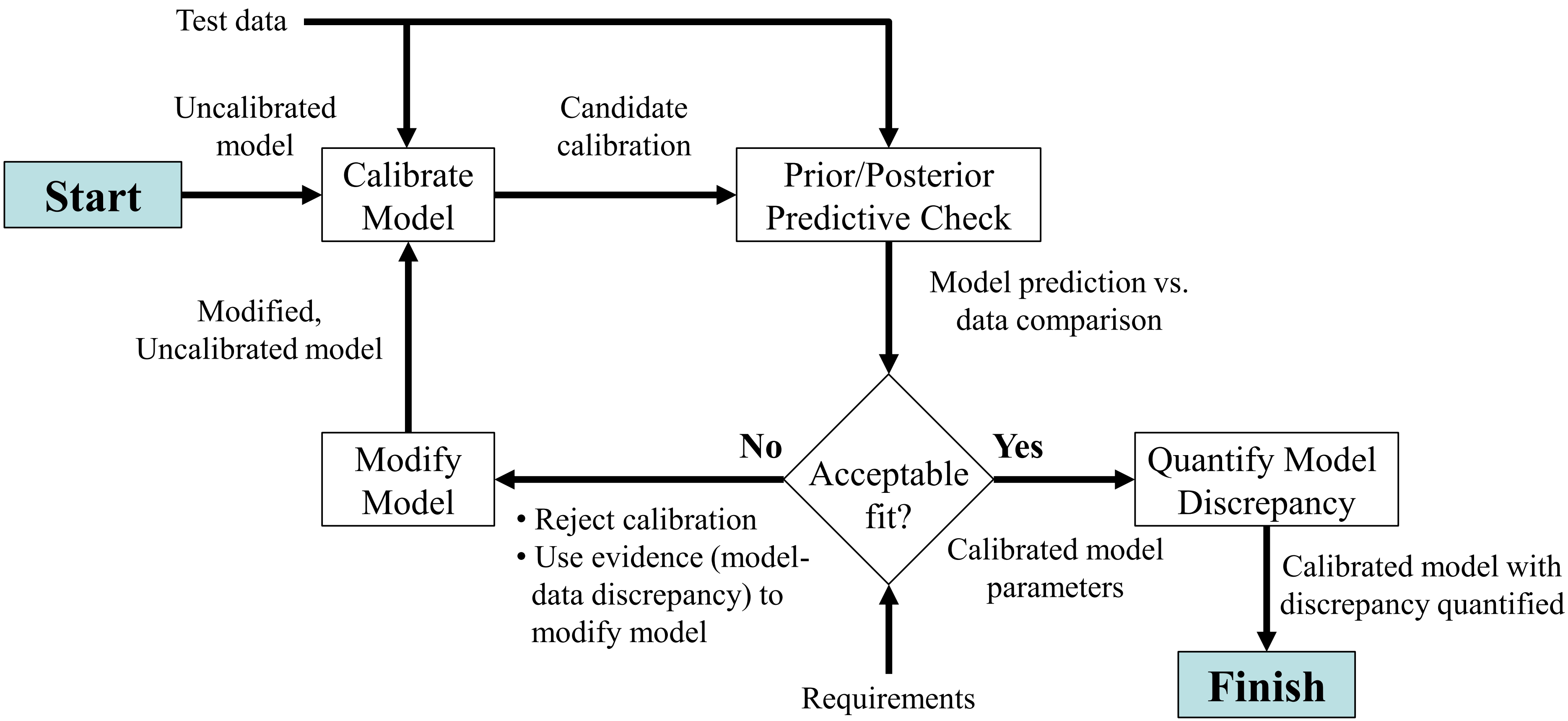
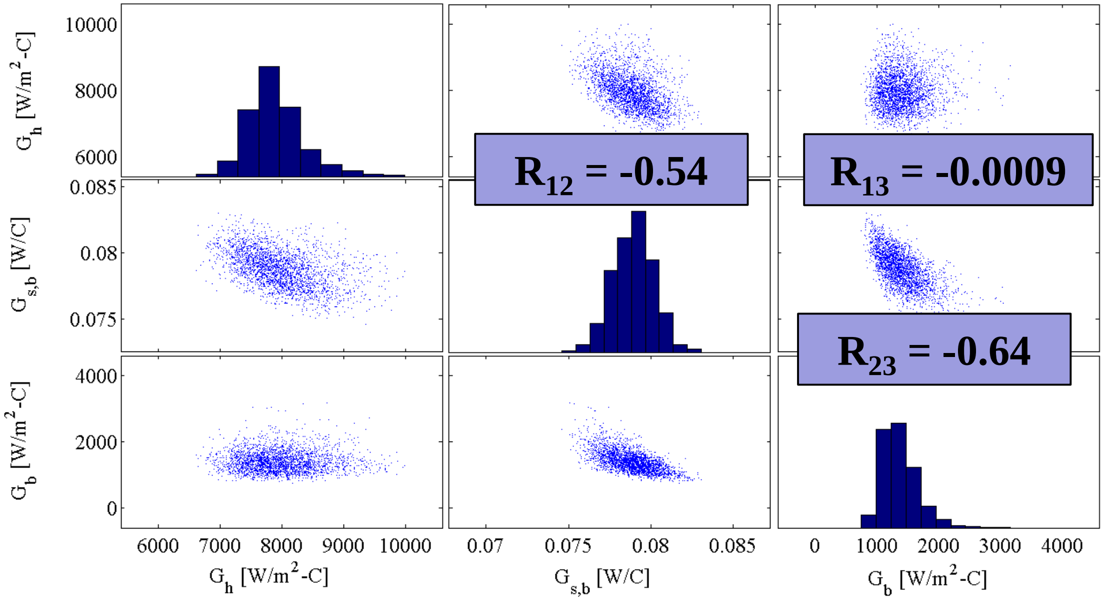


Model Calibration Process Overview



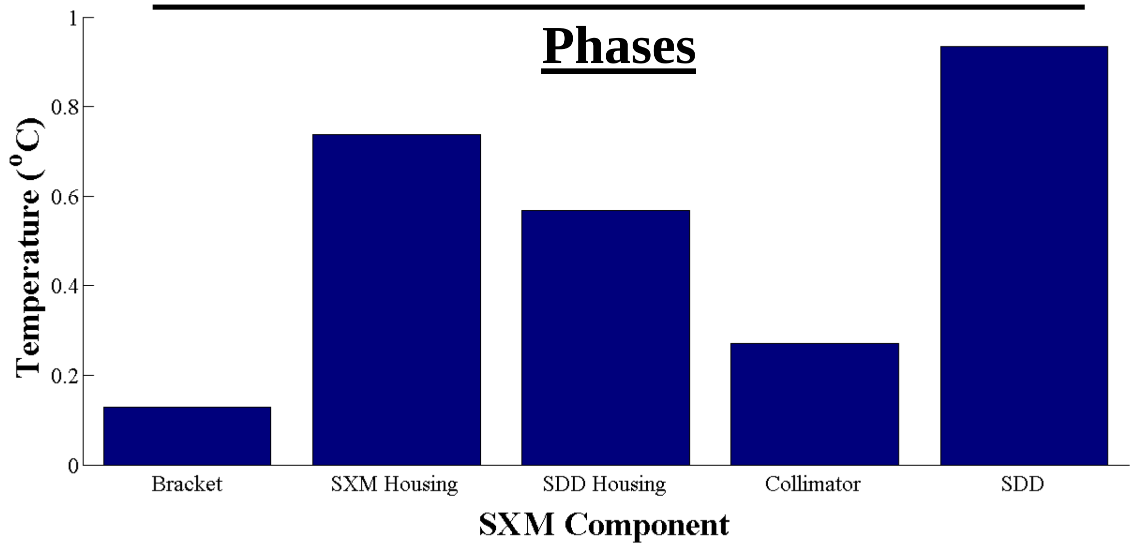
General process for calibration of model parameters and quantifying the model inadequacy

Markov Chain Monte Carlo Results



$$|\Delta T_{avg}| = \frac{1}{P}\sum_{i=1}^{P} |E[\eta_{SXM}(\mathbf{x}, \mathbf{d}_i)] - z_i|$$

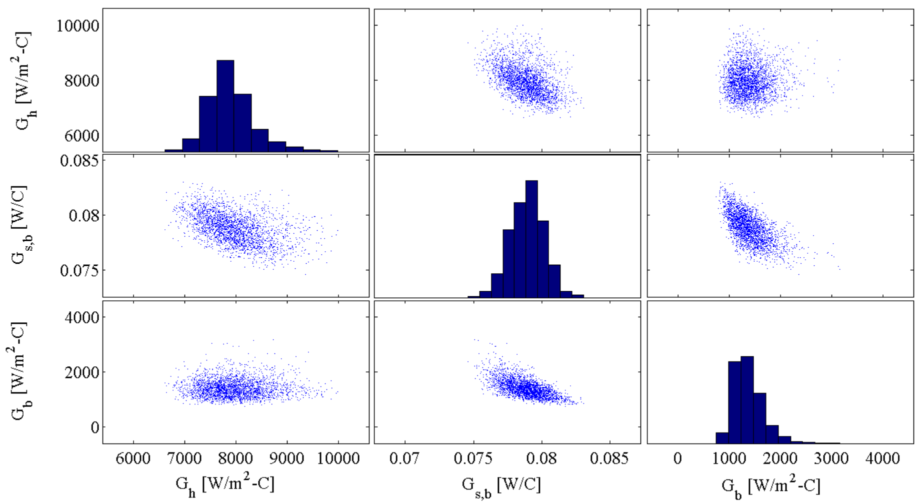
Posterior Predictive Check for all 43 Test Phases



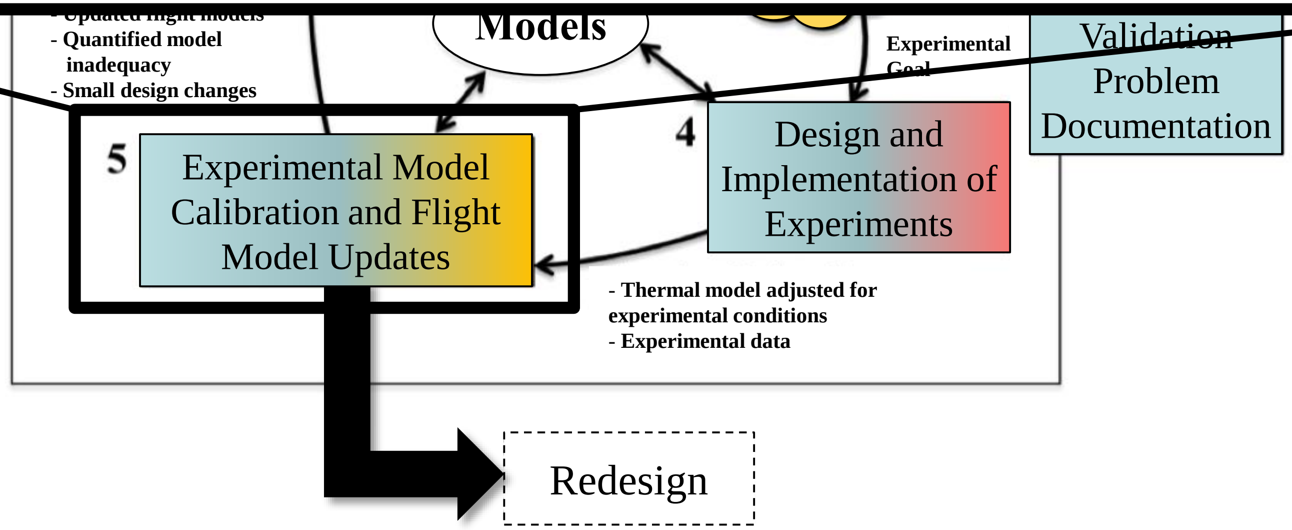
On average, difference between model prediction and data is less than 1 ^{o}C

Posterior parameter distributions yield acceptable fit to all data

Summary of SXM Case Study



- MCMC to calibrate parameters
- Persisting model discrepancy <1 °C, on average



Uncertainty Analysis

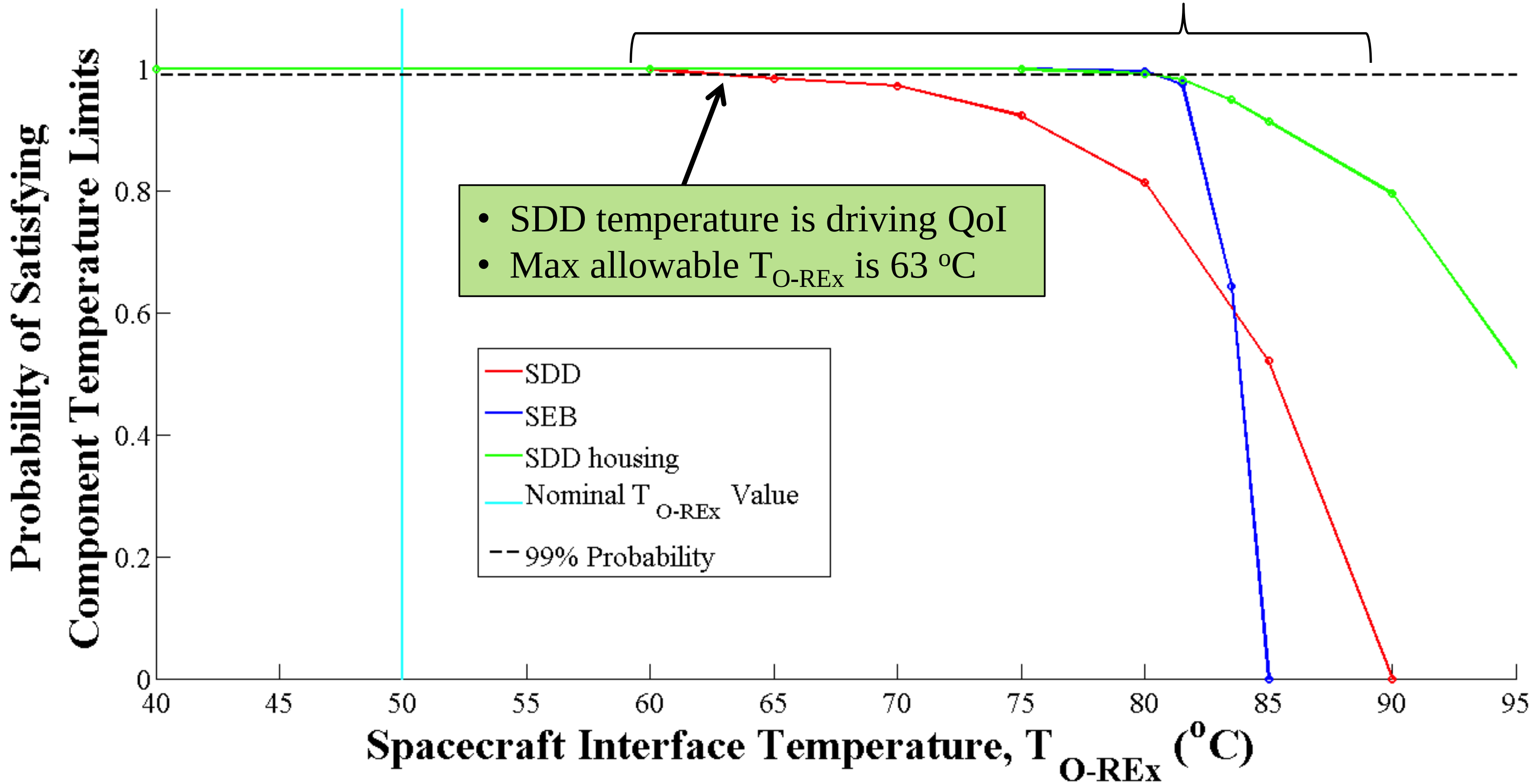
Monte Carlo (MC) Simulation

$$\overline{\eta}_{SXM,N} = \frac{1}{N}\sum_{i=1}^{N} \eta_{SXM}(\mathbf{x}_i)$$

x contains all system and environmental parameters

Parametric uncertainty results in uncertainty in max allowable $T_{O\text{-}REx}$

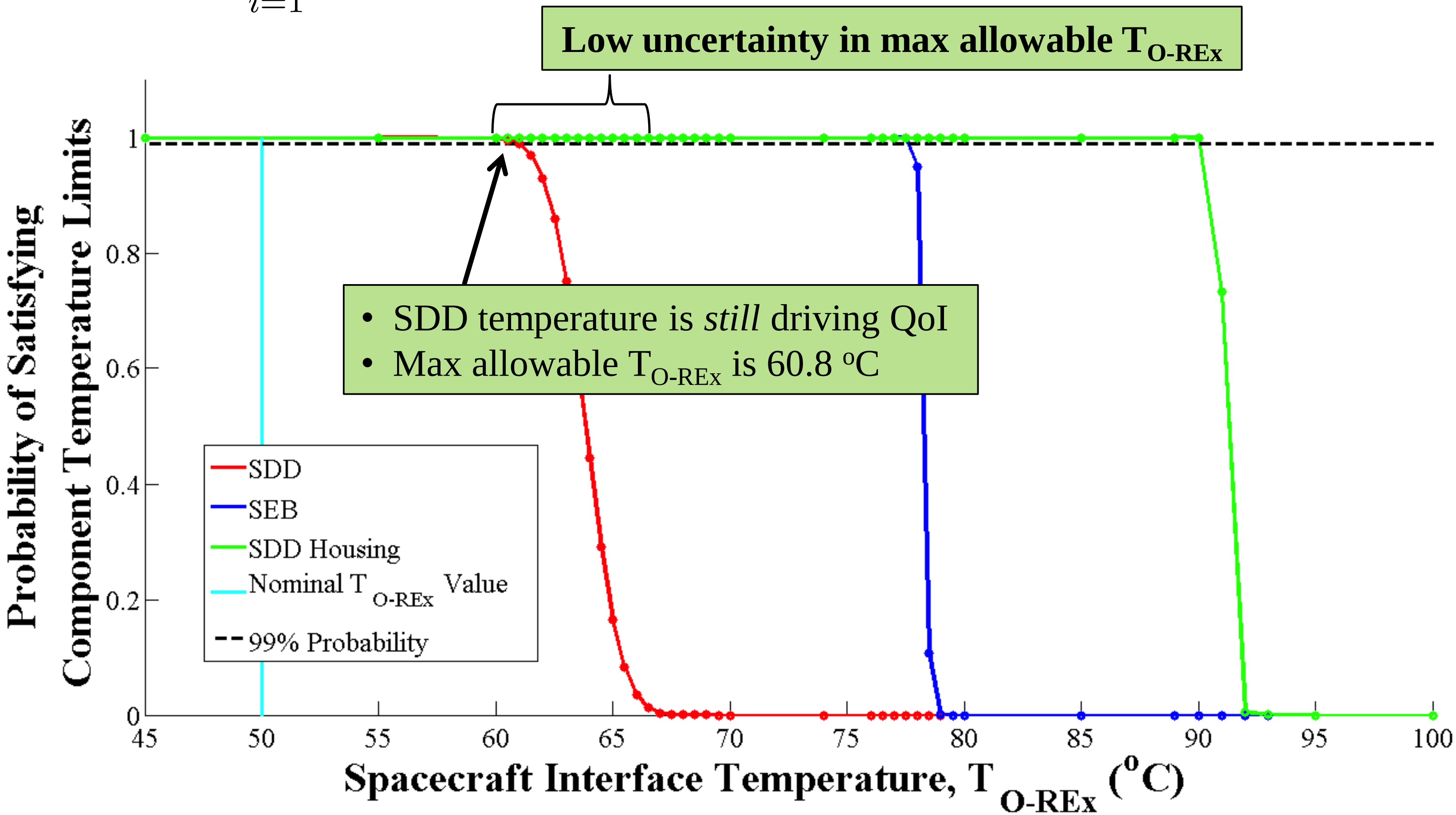
- SDD temperature is driving QoI
- Max allowable $T_{O\text{-}REx}$ is 63 ^{o}C



Updated Uncertainty Analysis

$$\zeta_{SXM}(\mathbf{x}) = \eta_{SXM}(\mathbf{x}) + \delta(\mathbf{d})$$ } True physical process, ζ_{SXM}

$$\overline{\zeta}_{SXM,N} = \frac{1}{N}\sum_{i=1}^{N}\zeta_{SXM}(\mathbf{x}_i)$$ } Monte Carlo simulation of true physical process



Summary of SXM Case Study

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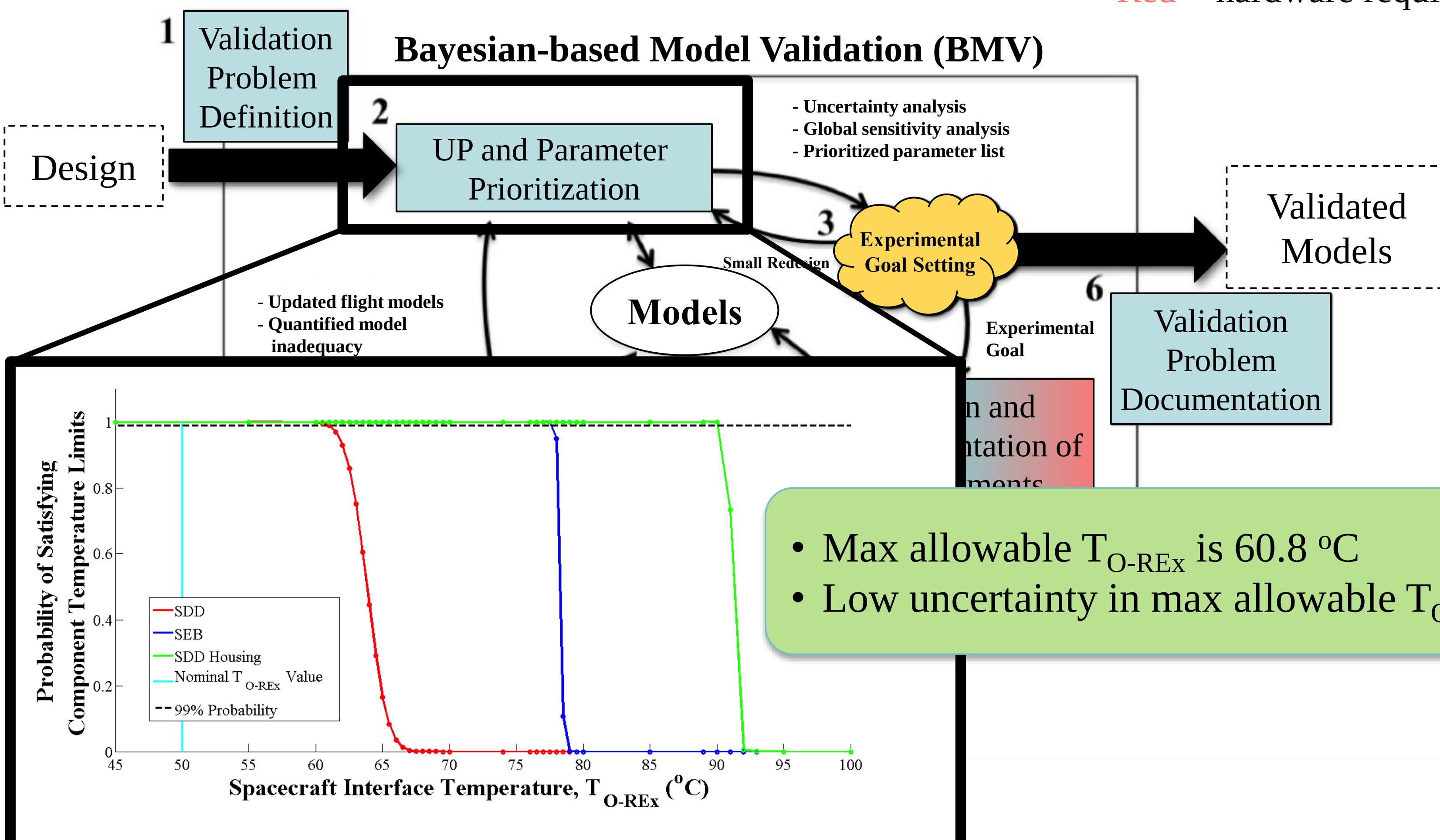
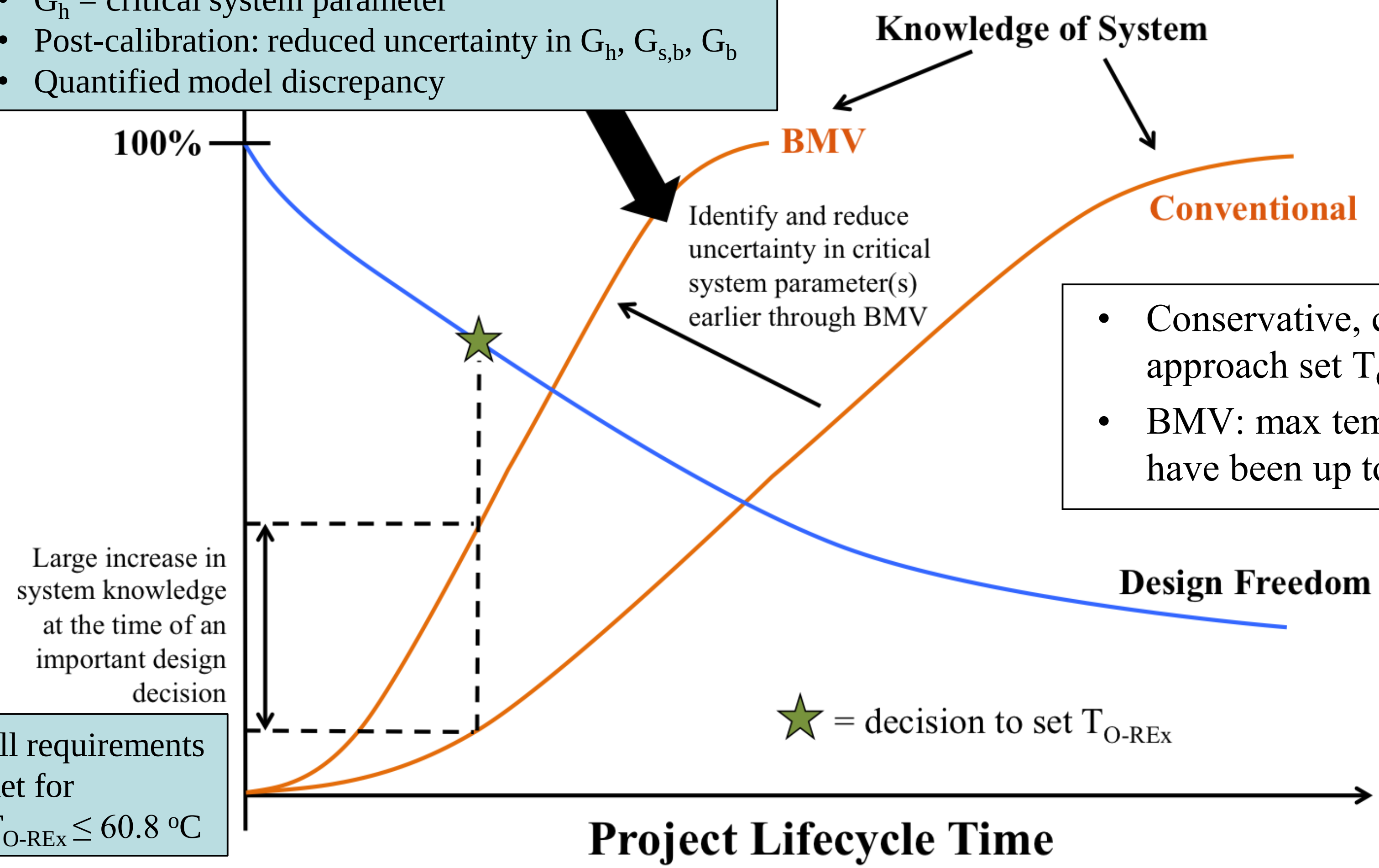


- Max allowable $T_{O\text{-}REx}$ is 60.8 ^oC
- Low uncertainty in max allowable $T_{O\text{-}REx}$

BMV Motivation – SXM Case Study

- G_h = critical system parameter
- Post-calibration: reduced uncertainty in G_h, $G_{s,b}$, G_b
- Quantified model discrepancy



- Conservative, conventional approach set $T_{O\text{-}REx}$ to 50 °C
- BMV: max temperature could have been up to 10 °C warmer

All requirements met for $T_{O\text{-}REx} \leq 60.8$ °C

Potential to increase knowledge of the system earlier in the project lifecycle when important design decisions are made

Conclusion

- Application of state of the art model uncertainty methods for thermal systems
- Created BMV methodology using state of the art UQ and DOE
- Implemented BMV on REXIS hardware
  - System level form and validation process improvements
- Future work:
  - Demonstrate BMV on larger, more complex thermal systems
  - Improve BMV interface with Thermal Desktop
  - Create databases of parameter uncertainty distributions

References – Background and Motivation

[1] D. Gilmore. Spacecraft Thermal Control Handbook, volume one. The Aerospace Corporation, El Segundo, CA, second edition, 2002.

[2] R. Karam. Satellite Thermal Control for Systems Engineers, volume 181. AIAA, 1998.

[3] R. Sargent. Verification and validation of simulation models. Proceedings of the conference on Winter Simulation Conference, 37, 2005.

[4] S. Blattnig, L. Green, J. Loukring, J. Morrison, R. Tripathi, and T. Zang. Towards a Credibility Assessment of Models and Simulations. AIAA, 49:2156–2172, 2008.

[5] G. Karpati, T. Hyde, H. Peabody, and M. Garrison. Resource Management and Contingencies in Aerospace Concurrent Engineering. AIAA SPACE 2012 Conference and Exposition, September 2012.

[6] V. Baturkin. Micro-satellites thermal control-concepts and components. Acta Astronautica, 56:161–170, 2005.

[7] T. D. Swanson and G. Birur. NASA thermal control technologies for robotic spacecraft. Applied Thermal Engineering, 23:1055–1065, 2003.

[8] J. Gardner, et al. The James Webb Space Telescope. Space Science Reviews, 123:485– 606, 2006.

[9] B. Allen, J. Grindlay, J. Hong, M. Oprescu, N. Induni, K. Fogarty, R.P. Binzel, R.A. Masterson, M. Chodas, N. Inamdar, M.J. Smith, G. Sondecker, M.W. Bautz, and J. Villasenor. The REgolith X-Ray Imaging Spectrometer (REXIS) for OSIRIS-REx: Identifying regional elemental enrichment on asteroids. SPIE Optical Engineering Applications. International Society for Optics and Photonics, 8840, August 2013.

[10] R.D. Stark. Thermal Testing of Spacecraft. Technical report, The Aerospace Corporation, 1971. Report No. TOR-0172(2441-01)-4.

[11] J. Welch. Comparison of Recent Satellite Flight Temperatures with Thermal Model Predictions. 36th International Conference on Environmental Systems (ICES), July 2006. Report No. 2006-01-2278.

[12] H. Peabody, J. Rodriguez-Ruiz, and V. Benitez. Thermal Margin Study for the Global Precipitation Measurement Spacecraft. 42nd International Conference on Environmental Systems, pages 3625–3636, July 2012.

References – Literature Review

[13] D. Allaire. Uncertainty Assessment of Complex Models with Application to Aviation Environmental Systems. PhD thesis, Massachusetts Institute of Technology, June 2009.

[14] J.C. Helton, J.D. Johnson, C.J. Sallaberry, and C.B. Storlie. Survey of sampling-based methods for uncertainty and sensitivity analysis. Reliability Engineering and Systems Safety, 91(10):1175–1209, 2005.

[15] Test Requirements for Launch, Upper-stage, and Space Vehicles. Department of Defense, 1999. MIL-HDBK-340A (USAF).

[16] Product Verification Requirements for Launch, Upper Stage, and Space Vehicles. Department of Defense, 1999. MIL-STD-1540D.

[17] General Environmental Verification Standard (GEVS). NASA Goddard Space Flight Center, 2013. GSFC-STD-7000A.

[18] R.A. Fisher. The Arrangement of Field Experiments, volume 33. 1926.

[19] R.A. Fisher. The Design of Experiments. Oliver and Boyd, London, England, second edition, 1935.

[20] A.C. Atkinson, A.N. Donev, and R.D. Tobias. Optimum Experimental Designs, volume 34. Oxford University Press, with SAS, Oxford, England, 2007.

[21] X. Huan and Y.M. Marzouk. Simulation-based optimal Bayesian experimental design for nonlinear systems. Journal of Computational Physics, 232(1):288–317, 2012.

[22] M.C. Kennedy and A. O'Hagan. Bayesian Calibration of Computer Models. Journal of the Royal Statistical Society, 63(3):425–464, 2001.

[23] J. Brynjarsdottir and A. O'Hagan. Learning about physical parameters: The importance of model discrepancy. SIAM/ASA Journal on Uncertainty Quantification, 2013.

[24] D. M. Higdon, J. Gattiker, B. Williams, and M. Rightley. Computer Model Calibration Using High-Dimensional Output. Journal of the American Statistical Association, 103(482):570–583, 2008.

[25] P. Z. G. Qian and C. F. J. Wu. Bayesian Hierarchical Modeling for Integrating Low- accuracy and High-accuracy Experiments. Technometrics, 50(2):192–204, 2008.

[26] M. J. Bayarri, J. O. Berger, R. Paulo, J. Sacks, J. Cafeo, J. Cavendish, C. Lin, and J. Tu. A Framework for Validation of Computer Models. Technometrics, 49(7):138–154, 2007.

[27] D. M. Higdon, M. C. Kennedy, J. C. Cavendish, J. A. Cafeo, and R. D. Ryne. Combining Field Data and Computer Simulations for Calibration and Prediction. SIAM Journal on Scientific Computing, 26(2):448–466, 2004.

References – SXM Case Study

[28] N. Metropolis, A.W. Rosenbluth, M.N Rosenbluth, A.H. Teller, and E. Teller. Equation of state calculations by fast computing machines. The Journal of Chemical Physics, 21(6):1087–1092, 1953.

[29] W.K. Hastings. Monte Carlo sampling methods using Markov chains and their applications. Biometrika, 57(1):97–109, 1970.

[30] S. Glasgow and K. Kittredge. Performance Testing of Thermal Interface Filler Materials in a Bolted Aluminum Interface Under Thermal Vacuum Conditions. Technical report, NASA Marshall Space Flight Center, 2003. Report NASA/TM-2003-212500.

Backup Slides

Posterior Sampling Formulation

Calibration Parameters

$$\gamma = [G_h,\ G_{s,b},\ G_b]^T$$

Bayes' Theorem

$$p(\gamma|\mathbf{z},\mathbf{x}) = \frac{p(\mathbf{z}|\gamma,\mathbf{x})p(\gamma)}{p(\mathbf{z}|\mathbf{x})}$$

Metropolis-Hastings Algorithm [28,29], method for Markov Chain Monte Carlo (MCMC)

1. **Draw proposal,** γ_{new}**, from** $q(\gamma_{new}|\gamma_{old})$
2. **Calculate acceptance ratio:**

$$\alpha(\gamma_{old},\gamma_{new}) = [1, \frac{\pi(\gamma_{new})q(\gamma_{old}|\gamma_{new})}{\pi(\gamma_{old})q(\gamma_{new}|\gamma_{old})}]$$

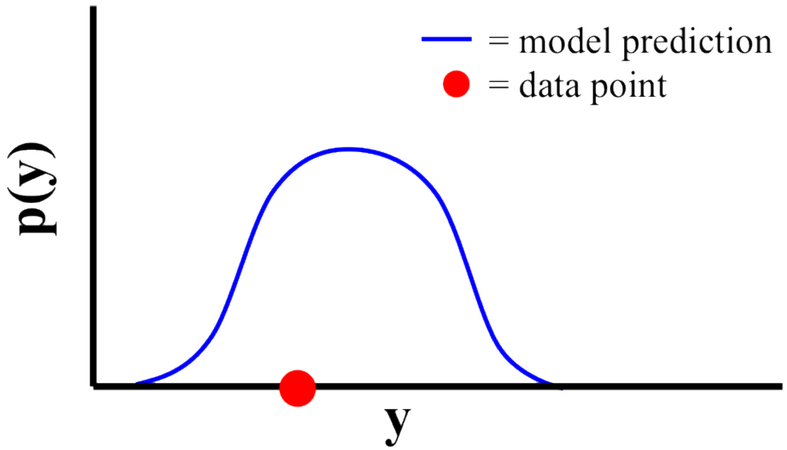
3. **Set the next value in the chain:**

$$\gamma_{n+1} = \begin{cases} \gamma_{new} & with\ probability\ \alpha(\gamma_{old},\gamma_{new}) \\ \gamma_{old} & with\ probability\ 1-\alpha(\gamma_{old},\gamma_{new}) \end{cases}$$

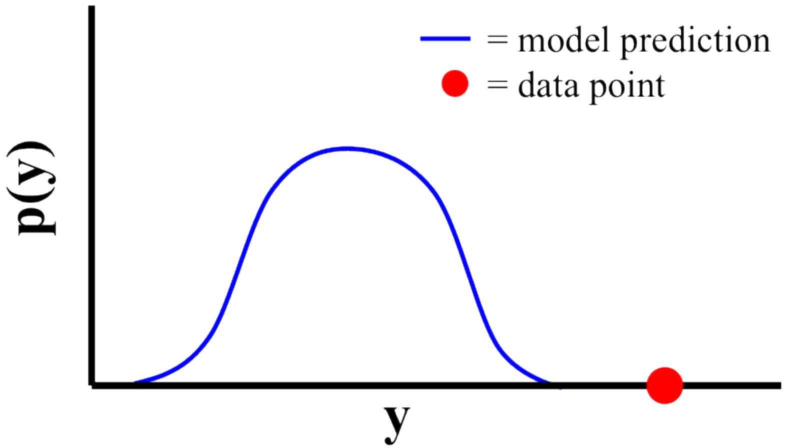
where $\pi(\gamma) = p(\gamma|\mathbf{z},\mathbf{d}) \propto p(\mathbf{z}|\gamma,\mathbf{d})p(\gamma))$

Bayesian inference: given the test data, MCMC is used to sample the posterior distributions of the calibration parameters

Posterior Check Explanation



(a) Plausible data

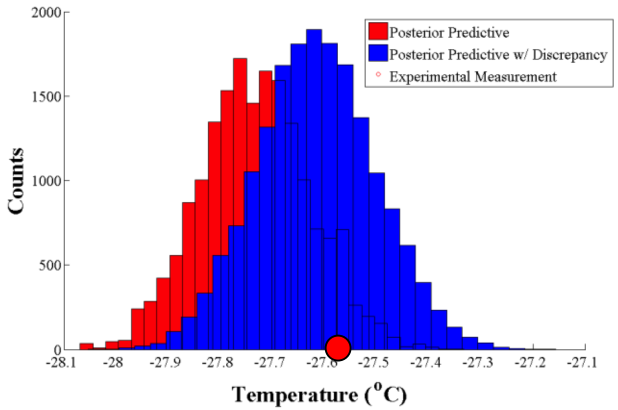


(b) Implausible data

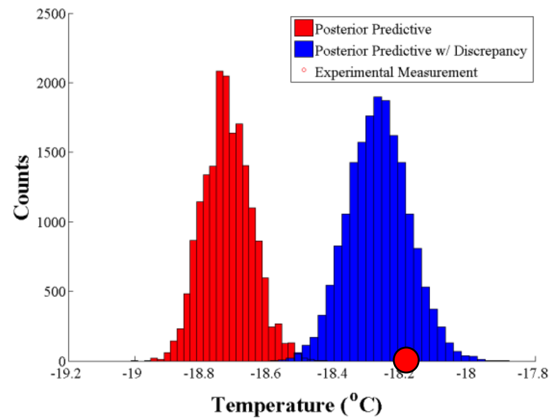
- **Calibration parameters:** $\gamma = [G_h \; G_{s,b} \; G_b]^T$
- All other parameters in **x** are fixed

Posterior Predictive Check for T36

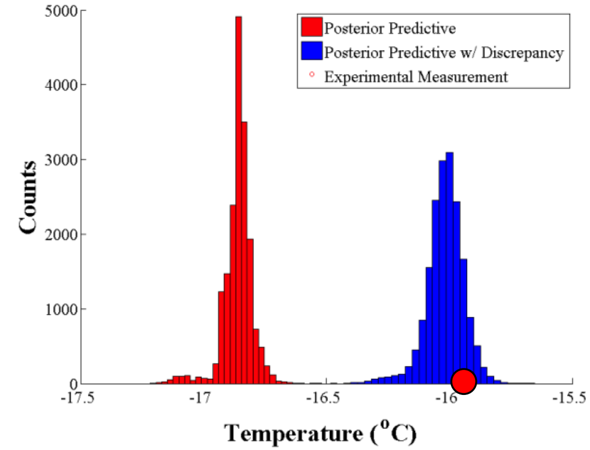
Bracket



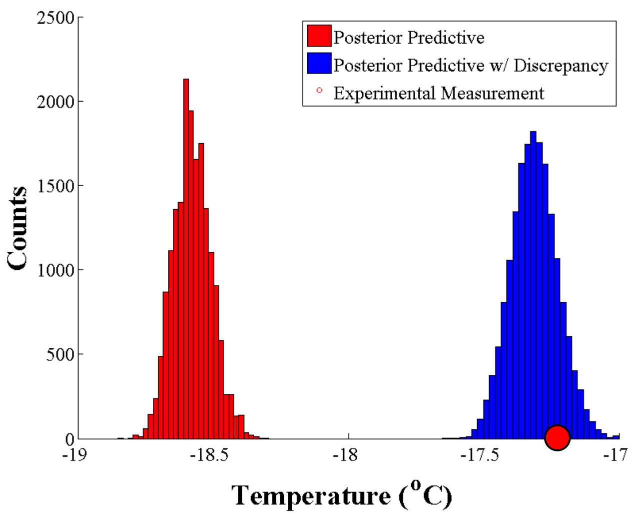
SXM Housing



SDD Housing



Collimator



Model discrepancy function improves model accuracy (all data plausible under model output)

Model Discrepancy Formulation

- Kennedy-O'Hagan formulation [22], additive model discrepancy
- Gaussian Process (GP) models
- Squared Exponential ARD covariance kernel

$$\underbrace{\delta(\mathbf{x})}_{\text{discrepancy}} = \underbrace{\mathbf{z}}_{\text{experimental observations}} - \underbrace{\eta_{SXM}(\mathbf{x}, \gamma)}_{\text{calibrated model}} - \underbrace{\epsilon_m}_{\text{observation error}}$$

function of **d** only

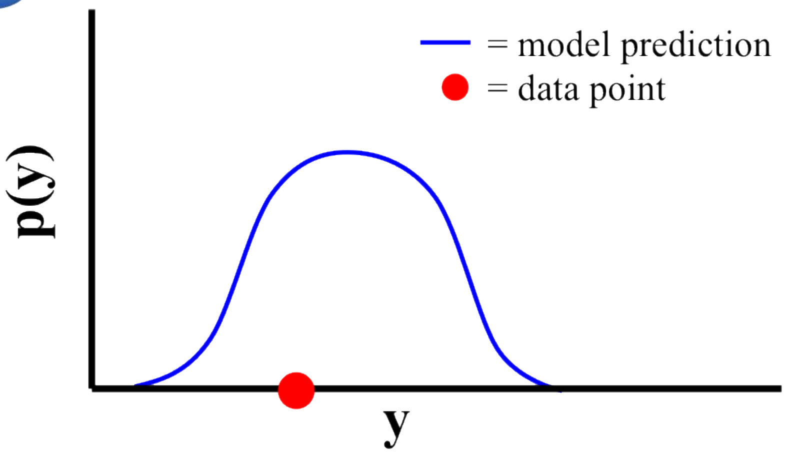
$$\delta(\mathbf{d}) = \underbrace{\begin{pmatrix} \delta_b \\ \delta_{sxm,h} \\ \delta_{sdd,h} \\ \delta_{coll} \\ \delta_{sdd} \end{pmatrix}}_{\text{5x1 vector corresponding to measurements on 5 SXM components}}$$

$$\delta(\mathbf{d}) \sim \underbrace{\mathcal{GP}(m(\mathbf{d}), k(\mathbf{d}, \mathbf{d}')) = \mathcal{GP}(0, k(\mathbf{d}, \mathbf{d}'))}_{\text{Zero-mean Gaussian Process, each discrepancy term is an independent function}}$$

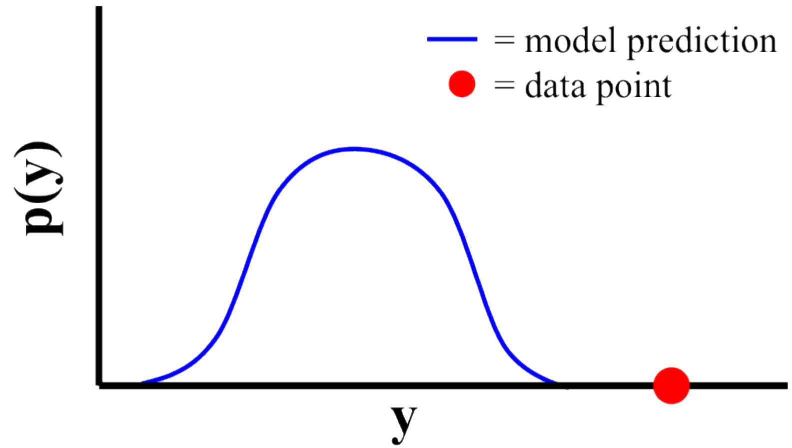
$$\underbrace{k(\mathbf{d}, \mathbf{d}') = \sigma_0^2 \ \ exp\left\{ -\frac{1}{2}\left(\frac{V_{TEC} - V'_{TEC}}{\lambda_1}\right)^2 + -\frac{1}{2}\left(\frac{T_{O-REx} - T'_{O-REx}}{\lambda_2}\right)^2 \right\}}_{\text{Squared exponential Automatic Relevance Determination (ARD) covariance kernel}}$$

GP models used to quantify the calibrated model discrepancy for all 43 test phases

Calibration Parameter Selection



(a) Plausible data



(b) Implausible data

Prior Predictive Check Sequence

G_h Only → G_h and $G_{s,b}$ → G_h and $G_{s,b}$, Relaxed Lower Bound → G_h, G_b, and $G_{s,b}$, Relaxed Lower Bound

- G_h Only ⇑ **SDD & SDD housing plausible**
- G_h and $G_{s,b}$ ⇑ **SDD & SDD housing plausible**
- G_h and $G_{s,b}$, Relaxed Lower Bound ⇑ **SDD, SDD housing, SXM housing, & collimator plausible**
- G_h, G_b, and $G_{s,b}$, Relaxed Lower Bound ⇑ **Data for all components plausible**

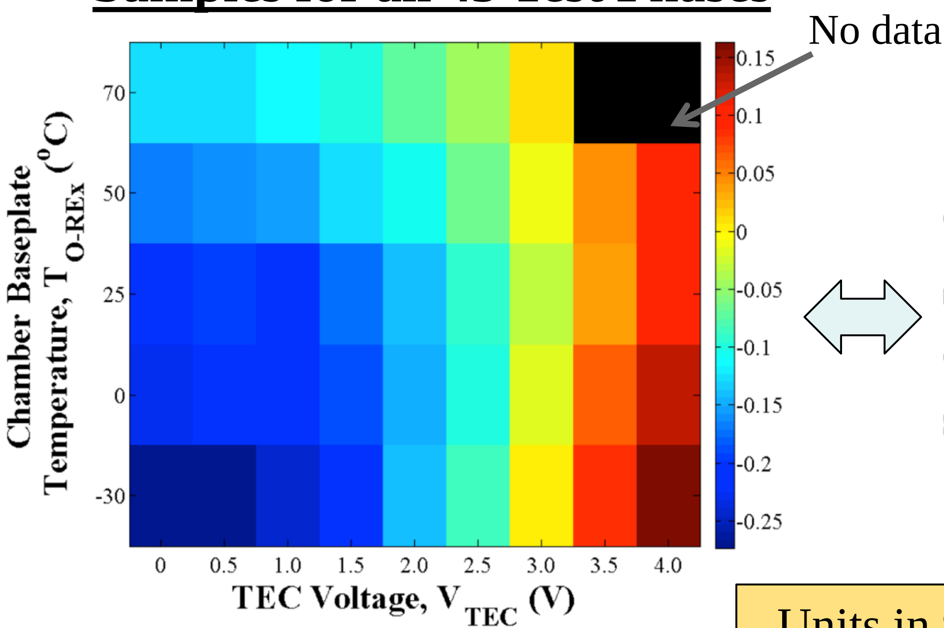
Calibration parameters: $\gamma = [G_h \; G_{s,b} \; G_b]^T$

All other parameters in **x** are fixed

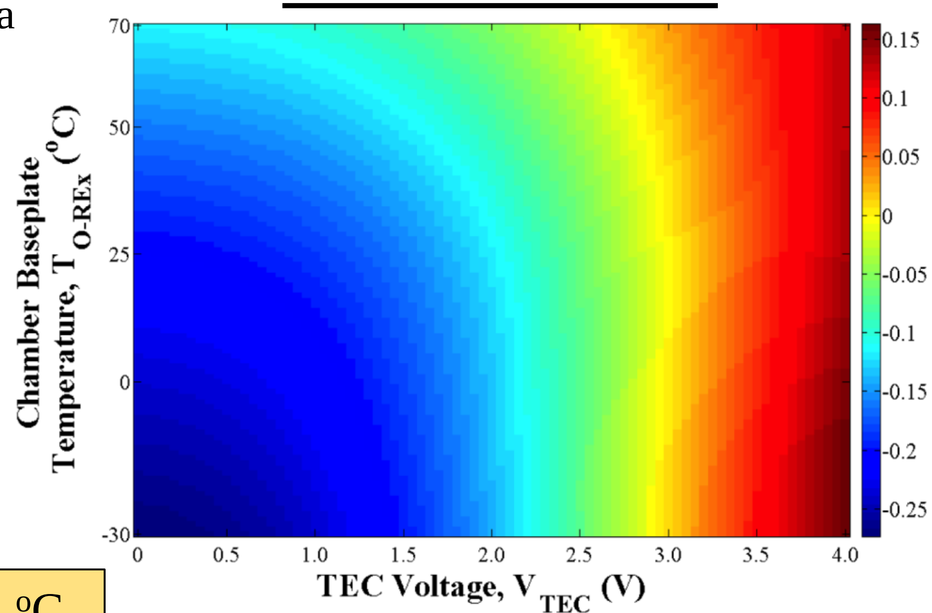
GP Model Regression Results

GP regression used to find hyperparameter values (σ_0, λ_1, λ_2) for each discrepancy function

Mean of Bracket Discrepancy Samples for all 43 Test Phases



Bracket GP Model Mean Value on Fine Grid



Units in ^oC

By inspection, regressed GP model mean is a good approximation of the mean of the discrepancy samples

Key Definition

Simulation Model Validation

Process of confirming a model is an adequate representation of the system and is capable of predicting the system's behavior *accurately with respect to requirements* over the <u>domain of the intended application</u> of the model [3,4]

Background – Thermal Simulation Models

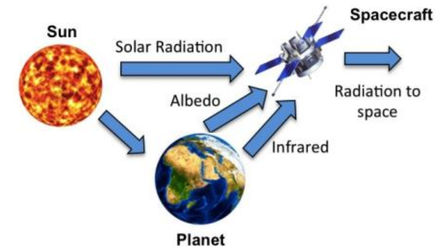
Parameters

System component geometry, connectivity, and material properties

Power dissipation of spacecraft components

Thermal Environment

General Spacecraft Thermal Environment



Model

General Heat Transfer Equation

$$\rho c_p \frac{\partial T}{\partial t} = \nabla \cdot k(\nabla \cdot T) + Q(T,t)$$

Symbol	Variable
ρ	Density
c_p	Specific heat
k	Conductivity
Q(T,t)	Source heat

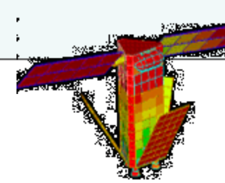
Thermal Analysis Tools

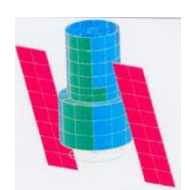
- Analytical models
- Lumped parameter models
- Commercially available software packages
  - TSS/SINDA and Thermal Desktop (Finite Difference)

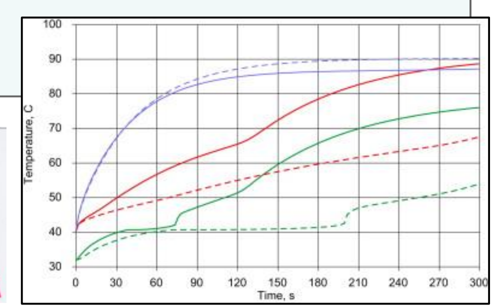
Increasing fidelity

Output

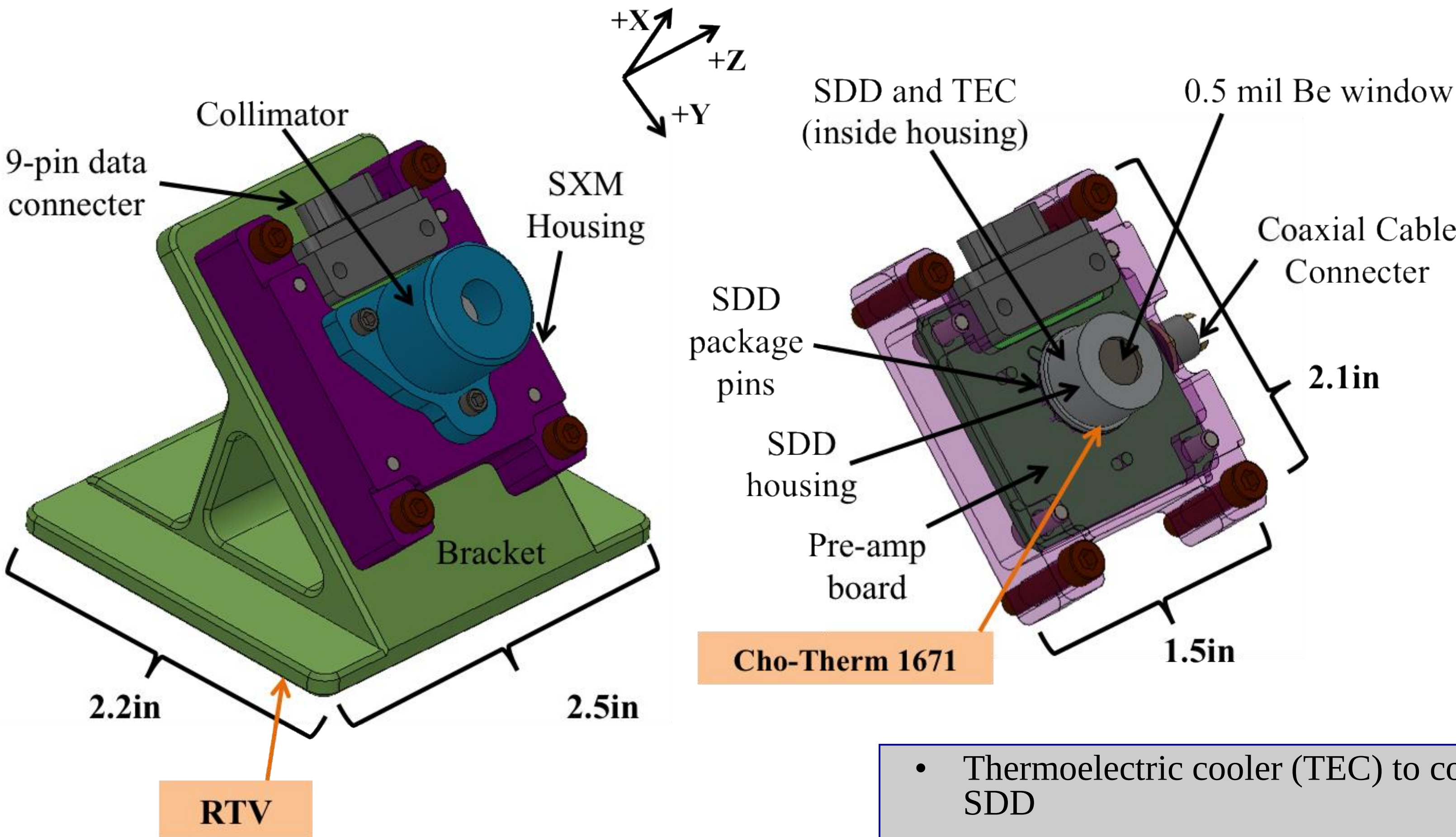
Predictions for spacecraft component temperatures for a given operational mode and thermal environment.







SXM Thermal Design Overview



- Thermoelectric cooler (TEC) to cool SDD
- Conduction dominates
- Thermally coupled to OSIRIS-REx interface, $T_{O\text{-}REx}$
- Nominally, $T_{O\text{-}REx}$ = 50 ºC

What is max allowable $T_{O\text{-}REx}$?

Experimental Set-Up

Example model modifications

– Include sensor/observation error, ε_m
– Thermal vacuum wall temperature is external radiation sink
– No sunlight
– TEC not software-controlled

Nomenclature

x: all model parameters

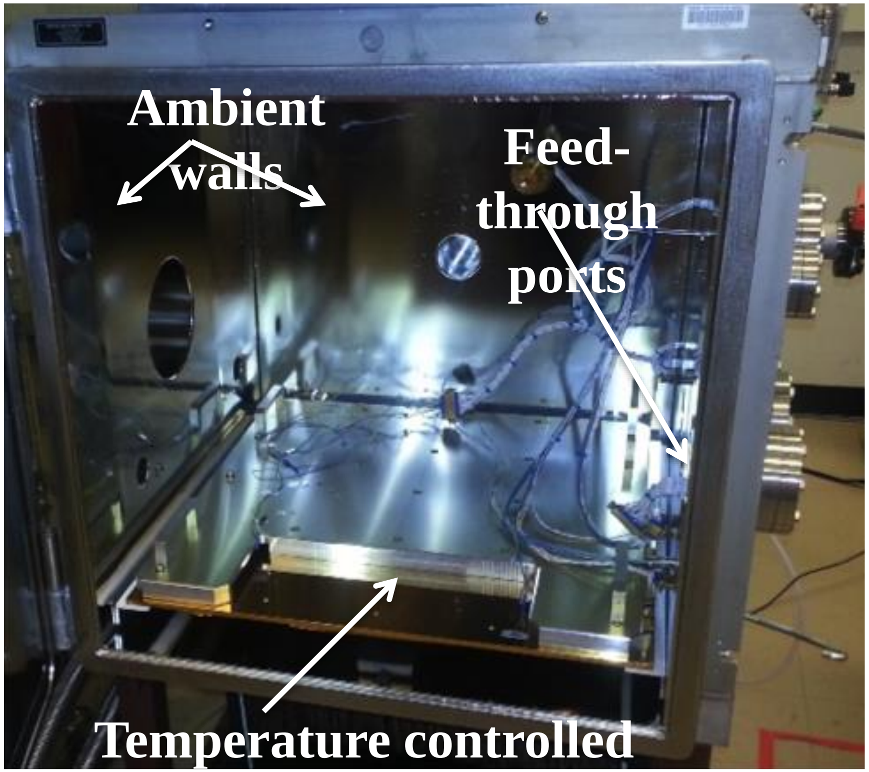
θ: parameter(s) of interest, $\theta = G_h$

d: experimental conditions,

$\mathbf{d} = [T_{O\text{-}REx}, V_{TEC}, T_w]^T$

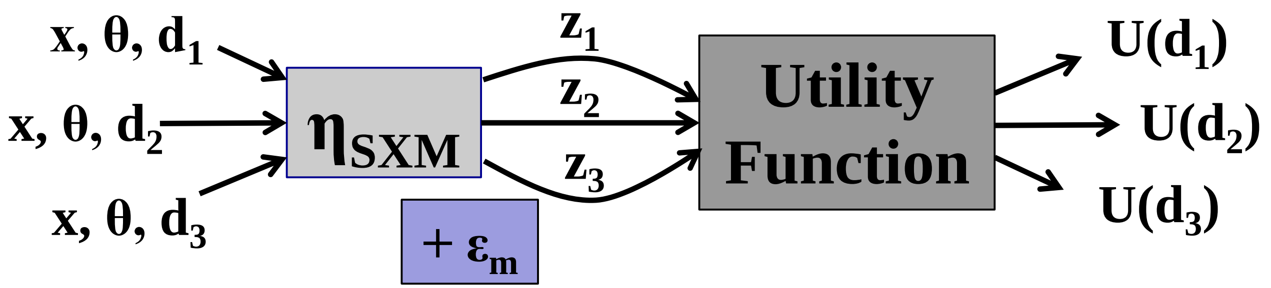
z: experimental result/data

SSL Thermal Vacuum Chamber



SSL chamber used for both parameter inference and model validation experiments

Parameter Inference Experiment



The Kullback-Leibler (KL) divergence utility function:

$$u(\mathbf{d}, \mathbf{z}, \theta) = D_{KL}(p(\theta|\mathbf{z}, \mathbf{d})||p(\theta))$$

inserted into Lindley's *expected* experimental utility form [21]:

$$U(\mathbf{d}) = E[D_{KL}(p(\theta|\mathbf{z}, \mathbf{d})||p(\theta))]$$

$$U(\mathbf{d}) \approx \frac{1}{n_{out}} \sum_{i=1}^{n_{out}} \Big(ln[p(\mathbf{z}_i|\theta_i, \mathbf{d})] - ln[p(\mathbf{z}_i|\mathbf{d})] \Big)$$

$$p(\mathbf{z}_i|\mathbf{d}) \approx \frac{1}{n_{in}} \sum_{j=1}^{n_{in}} p(\mathbf{z}_i|\theta_{i,j}, \mathbf{d})$$

$$\mathbf{d}^* = \max_{\mathbf{d} \in \mathcal{D}} U(\mathbf{d})$$

Background – Bayesian Probability

- **Interpretation of probability:** instead of quantifying "frequency" or "propensity," a Bayesian probability is a quantity defining a state of knowledge
- **Bayesian inference**
  - Given new information, the probability is updated via Bayes' Theorem
- **Broadly applicable to many engineering disciplines**
  - "Natural" fit to many engineering problems
  - "Common sense" interpretation of statistical conclusions

Bayes' Theorem

$$p(\gamma|\mathbf{z},\mathbf{x}) = \frac{p(\mathbf{z}|\gamma,\mathbf{x})p(\gamma)}{p(\mathbf{z}|\mathbf{x})}$$

Comparison of Probability Interpretations*

Frequentists	Bayesians
Probabilities represent long term frequencies of repeatable random experiments	Probabilities describe the incomplete knowledge of a fixed parameter or quantity
Data are repeatable, random sample	Data observed from realized sample
Unknown parameters are constant Parameters are fixed	Parameters are unknown and described probabilistically Data are fixed

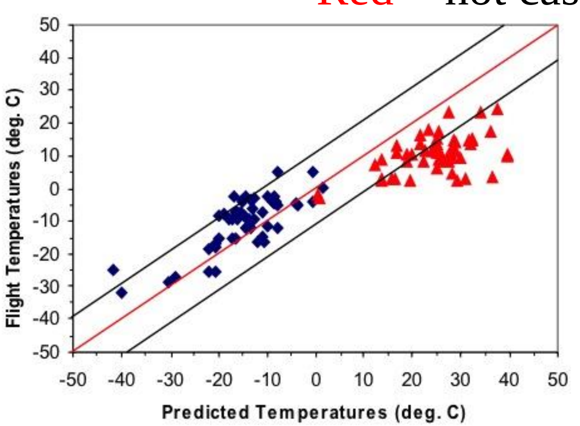
*Casella, George. "Bayesians and Frequentists." ACCP 37th Annual Meeting, Philadelphia, PA. Department of Statistics, University of Florida.

Motivation – Evidence* (Welch 2006)

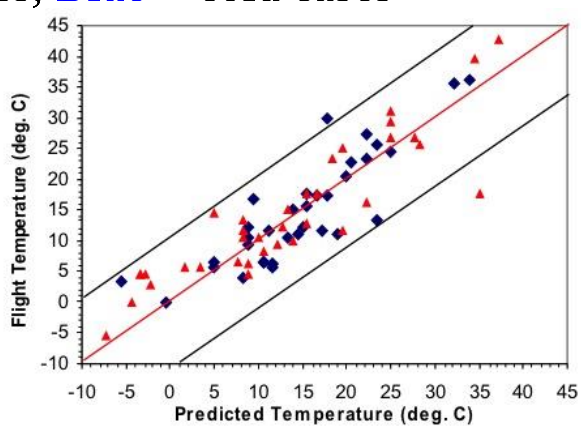
Flight Program	Model vs. Flight Temperature Difference $\mu \pm 2\sigma$ (°C)	Derived Thermal Uncertainty Margin (°C)
DOD Program A	$+5.9 \pm 10.0$	15.9
DOD Program B	$+1.3 \pm 8.4$	9.7
Iridium	-3.3 ± 11.9	15.2
NASA TIMED	$+4.3 \pm 11.2$ (cold) -13.5 ± 15.6 (hot)	15.5 29.1
DOD Program C	$+6.6 \pm 9.0$	15.6
DOD Program D	$+0.5 \pm 10.0$	10.5
ESA Italsat-1	$+2.2 \pm 7.8$	10.0
ESA Italsat-2	-1.5 ± 7.7	9.2
ESA SAX	-3.1 ± 6.6	9.7

- Revisited military standards for uncertainty margin
- Examined variety of programs, e.g. military, NASA, and ESA programs

Red = hot cases, Blue = cold cases



(a) NASA TIMED



(b) DOD Program D

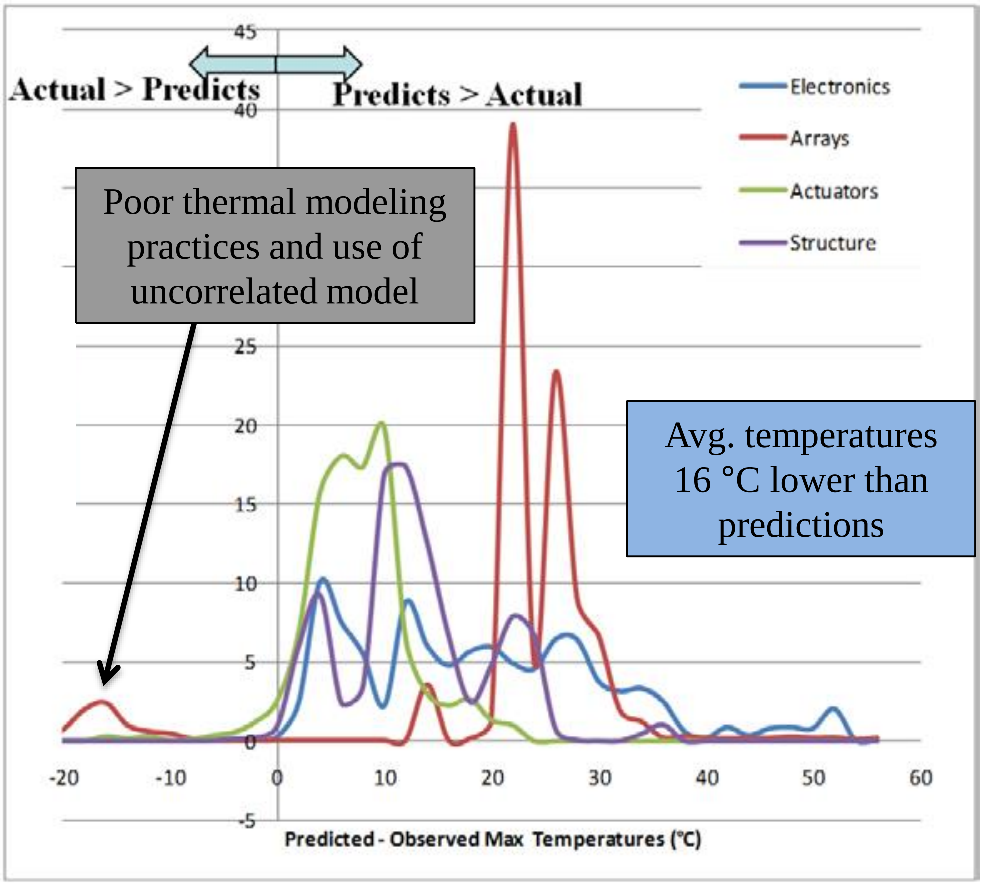
(a) **NASA TIMED**
- Very biased thermal model
- Intermediate environments significantly more benign than worst-case hot scenario

(b) **DOD Program D**
- Very little model bias, i.e. mean near zero
- Large variance about hot/cold case mean

*All data, tables, and figures from Welch [11]

Motivation – Evidence* (Karpati et al. 2012)

Flight temperatures vs. model predictions for seven recent GSFC missions**



*All data from Karpati, et al. [5]
**Daily/orbit max temperatures polled for 209 sensors for entire life of missions.

- Nearly all worst hot case predicted temperatures greater than those observed
  - Results agree with Welch [11] and Peabody, et al. [12]
  - Evidence that stacked worst case scenarios have low likelihood/frequency
- Estimated that the 5 °C NASA uncertainty margin [17] will result in radiator mass growth between 0.3-0.7 kg per 100 W heat load
  - Radiator growth leads to power draw increase of 4-6 W per 100 W heat load for survival heaters

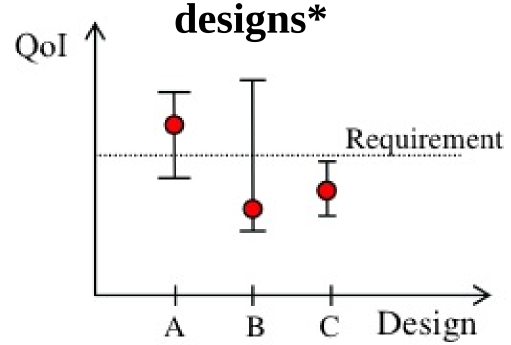
Uncertainty Propagation (UP)

State of the Art

Uncertainty Propagation Process [13]

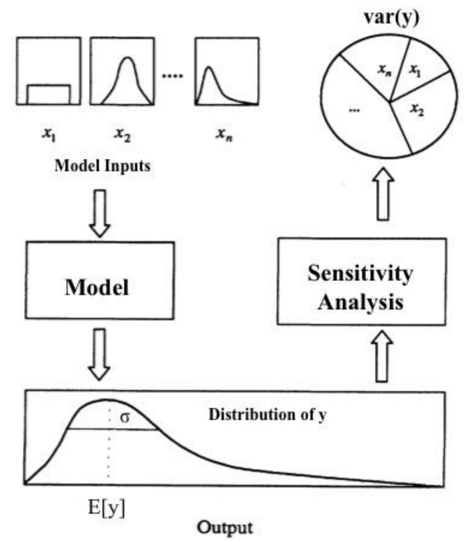
- Goal setting
- Model selection and documentation
  - Surrogate modeling
- Uncertainty classification
- Uncertainty characterization
- Uncertainty Analysis (UA)
- Sensitivity Analysis (SA)

UA for three different designs*



*S.A. Uebelhart, D. Miller, and C. Blaurock. Uncertainty Characterization in Integrated Modeling. AIAA Structures, Structural Dynamics and Materials Conference, 46:2005– 2142, April 2005.
**A. Saltelli, S. Tarantola, and K.P.-S. Chan. A Quantitative Model-Independent Method for Global Sensitivity Analysis of Model Output. Technometrics, 41(1):39–56, 1999. American Statistical Association and the American Society for Quality.

Global SA concept**



Thermal Convention

Most programs follow the philosophy in NASA GOLD Rules [17]:

Rule: Use model to show adequate margin between component temperature limits and stacked worst case temperature predictions.

Rationale: Positive margins account for uncertainties in power dissipations, environments, and thermal system parameters.

Stacked worse case scenarios [1]

- Heat loads
- Coating degradations
- Power dissipations
- Beta angles
- Critical conductances
- MLI e*

Design of Experiments (DOE)

State of the Art

Classical DOE
- Ronald Fisher [18,19]
  - Est. null hypothesis
- Principles of DOE
  - Randomization, blocking, replication, orthogonality
- No unified strategy and predefined experiments for general system

Optimal Bayesian Experimental Design (OBED)
Culminate to Huan and Marzouk [21]:

Bayesian statistics offer inference from noisy, indirect, and incomplete data.

- *Update* prior parameter distributions to reduce uncertainty
- Framework allows for *different experimental goals*, e.g. parameter inference
- Measure utility based on experimental result
  - Utility function based on predictive variance or parameter of interest, e.g. Kullback-Leibler divergence

Thermal Convention
- Models validated through thermal balance testing
- Classical DOE approach (same testing philosophy):
  - NASA – GEVS [17]
  - Military – MIL-STD-1540 [15] and MIL-HDBK-340 [16]
  - Other, e.g. universities

What cases and *how* a system should be tested to achieve model validation
- Test levels
- Environmental conditions
- Duration

Min Requirement: Two test conditions shall be imposed: one each at mission hot and cold case. NASA engineers shall select one additional case, per GEVS.

Primary Objective: Validate the design/model, which will be used to make predictions for the entire range of modes/mission environments.

Model Calibration

State of the Art

Parameter optimizations
- Cullimore [#]
- Masterson [#]

Focuses only on parameters

Bayesian Calibration
Seminal Paper – Kennedy and O'Hagan [22]
- General Bayesian calibration framework
- Non-linear, black box models
- Captures all parametric and non-parametric uncertainties
- Model inadequacy quantified after experiment

K-O Approach Enhancements
- Brynjarsdottir and O'Hagan [#]
  - Model the model inadequacy
- Higdon et al. [#]
  - High dimensional output
- Bayarri et al. [#]
  - Model validation framework

Thermal Convention

Correlation process outlined by Gilmore [1] followed for most space-based thermal systems:

1. Configure model based on environment and power modes tested

2. For a single test phase, adjust model to match data. Common adjustments to the model include:
   - Physical model omissions, i.e. model inadequacy
   - View factor geometries
   - Conductances
   - Power dissipations

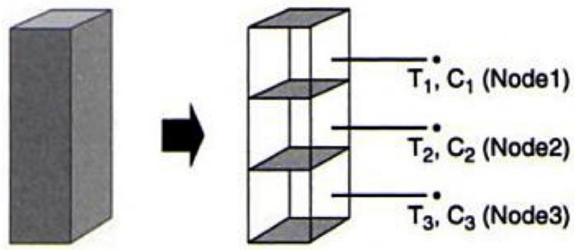
3. Correlate all temperature differences between model and test data to less than some threshold value, e.g. ±3°C per MIL-HDBK-340 [16]

4. Repeat 2-3 for the remaining test phases, ensuring that changes made in each remaining phase do not undo the correlation from a previous phase

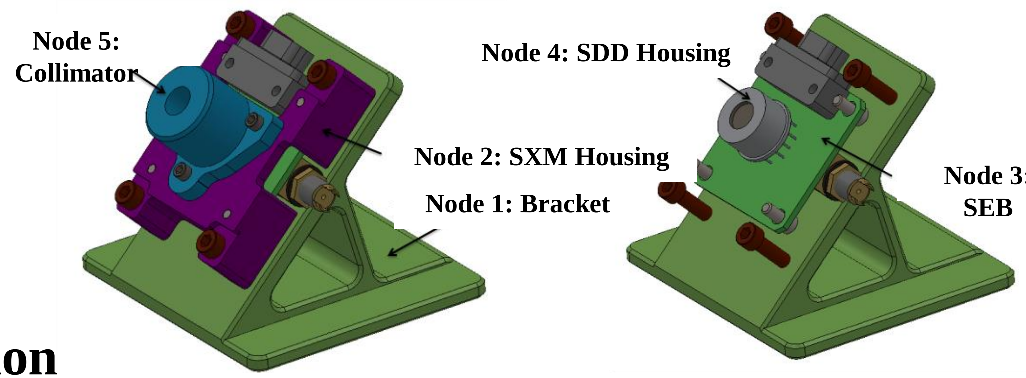
Ad hoc search for best fitting model parameters: relies heavily on engineering experience and intuition.

Model Formulation

Lumped Parameter Concept [1]



SXM Lumped Parameter Nodes



Model Formulation

$\mathbf{y} = \eta_{SXM}(\mathbf{x})$ where the three QoIs are identified in the output $\mathbf{Q} \subset \mathbf{y}$ and $\mathbf{Q} = [T_h, T_{pa}, T_{sdd}]^T$

Governing differential equation of form: $\frac{dT}{dt} = f(T, t)$ where the nodal temperatures are $T = [T_1 \; T_2 \; \ldots \; T_n]^T$

Simplifying Assumptions:

- Heterogeneous material globally, but the material assigned to each node is homogenous
- All SXM material is isotropic
- All material within a nodal region is isothermal

$$f(\mathbf{T}, t) = \mathbf{C}^{-1}[\mathbf{G}\mathbf{T} + \mathbf{Q}(\mathbf{T}, t)]$$

$$T_j(t_{i+1}) = \Delta t \left.\frac{dT}{dt}\right|_{T_j, t_i} + T_j(t_i)$$

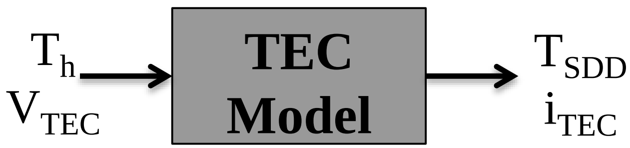
Select Δt such that solver is stable and has acceptable error

$$\mathbf{C} = \begin{pmatrix} m_{1,1}c_{p1,1} & 0 & \cdots & 0 \\ 0 & m_{2,2}c_{p2,2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & m_{n,n}c_{p3,3} \end{pmatrix}$$

$$\mathbf{G} = \begin{pmatrix} G_{1,1} & G_{1,2} & \cdots & G_{1,n} \\ G_{2,1} & G_{2,2} & \cdots & G_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ G_{n,1} & G_{n,2} & \cdots & G_{n,n} \end{pmatrix}$$

Prior SXM TEC Model

Thermoelectric Cooler (TEC)

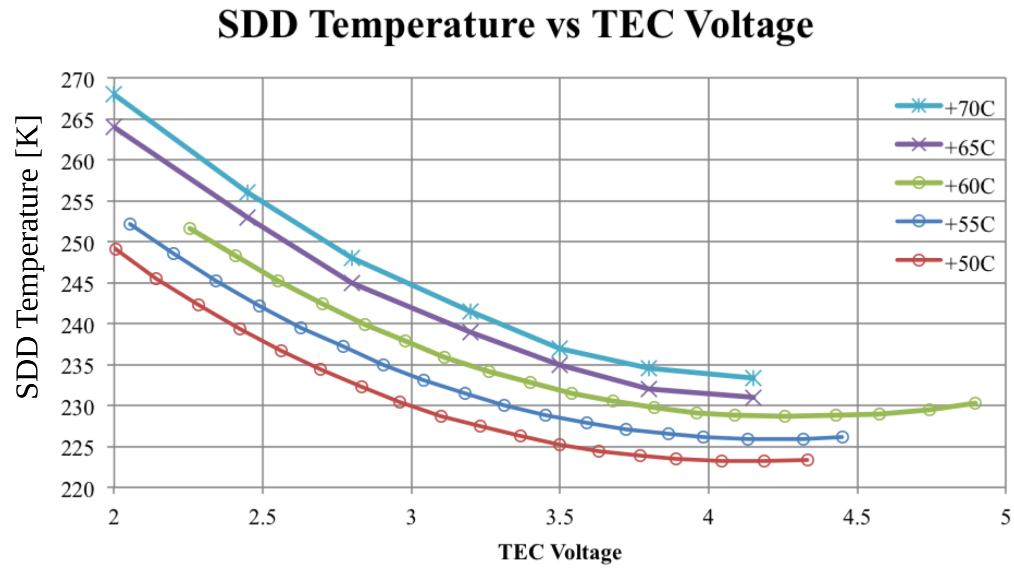


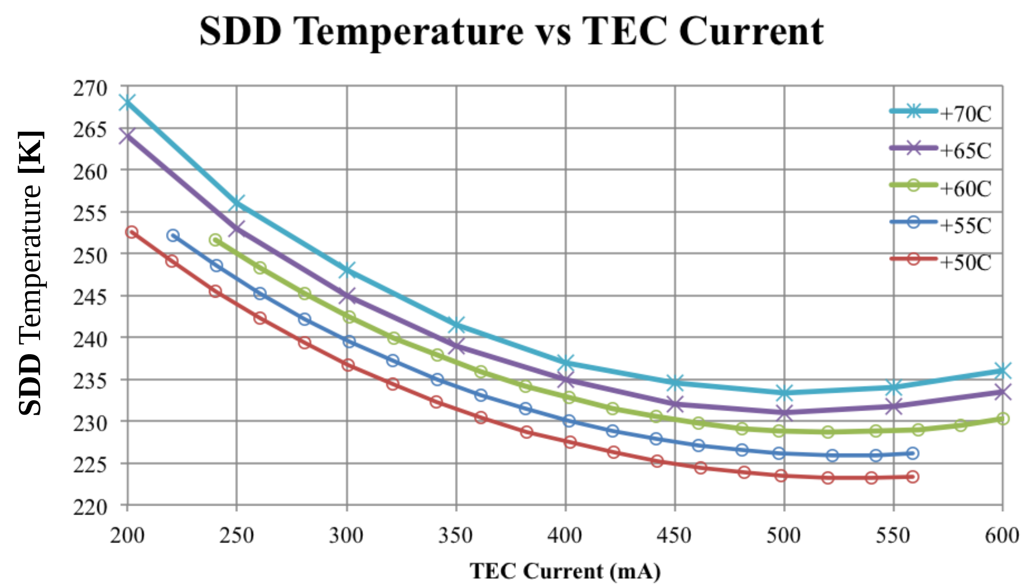
Proportional Control to Setpoint, T_s

$$V(t_{i+1}) = K_p e(t) = K_p(T_{sdd} - T_s)$$

- Performance estimates provided by Amptek, Inc. used to predict SDD temperature
- Parameters of polynomial curves are fixed values
- In flight, V_{TEC} will be controlled by flight software
- As the hot side temperature, T_h, increases, more power is required

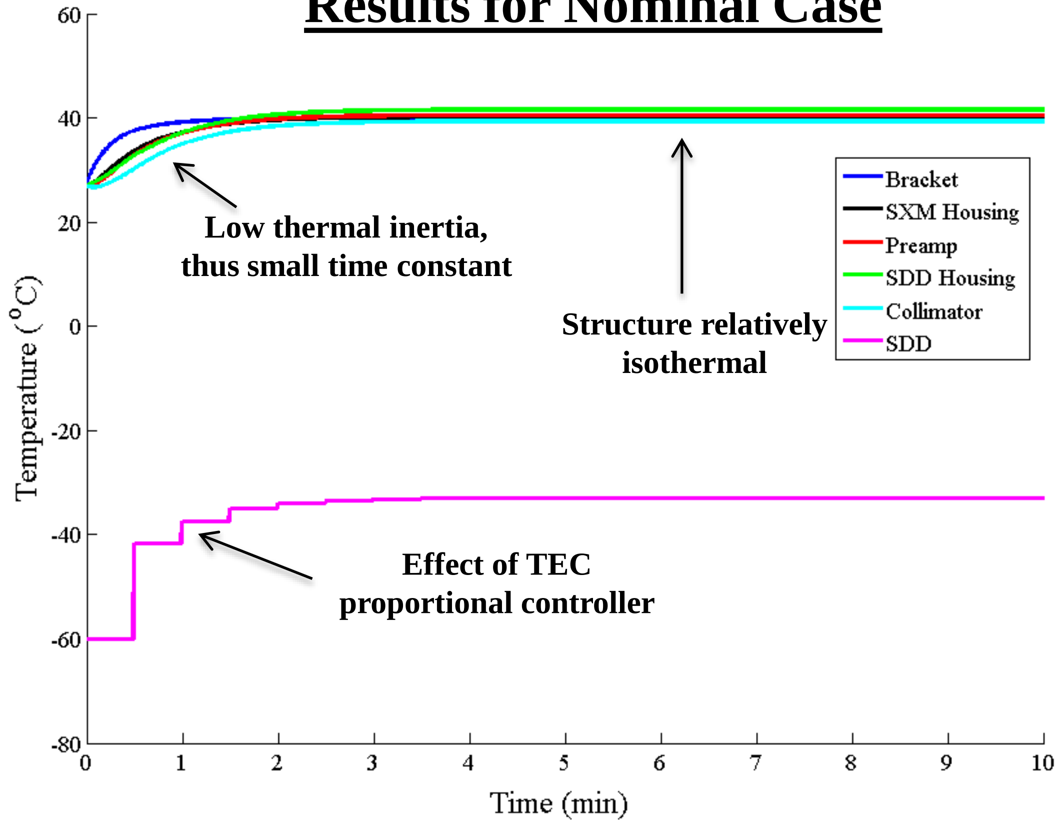
Thermal analysis uses TEC model and controller to focus on *the ability of the TEC to achieve T_s = -30°C.*





SXM Model Nominal Parameters

Results for Nominal Case



- 38 total parameters
  - 18 are uncertain or naturally exhibit variation
- What is meant by nominal?
  - Default design value
  - Current best estimate
  - Median parameter value

Requirements

Component	Operational (°C)	
	Min	Max
SDD Housing	-40	100
SEB	-40	85
SDD	-100	-30

Nominally, all three steady-state temperature requirements are satisfied

SXM Model Uncertain Parameters

Parameter Number	Name	Variable	Units	Nominal Value	Distribution Type	Parameter 1 (minimum)	Parameter 2 (maximum)
Node Specific Heats							
1	1 - Bracket	$c_{p,1}$	J/kg-K	961	Uniform	921	972
2	2 – SXM Housing	$c_{p,2}$	J/kg-K	961	Uniform	921	972
3	3 – Pre-amp board	$c_{p,3}$	J/kg-K	800	Uniform	378	880
4	4 – SDD Housing	$c_{p,4}$	J/kg-K	461	Uniform	378	461
5	5 - Collimator	$c_{p,5}$	J/kg-K	961	Uniform	921	972
Power Dissipations							
6	SDD	Q_{SDD}	W	0.01	Uniform	0	0.01
7	Pre-amp board	Q_{PA}	W	0.20	Uniform	0	0.25
Conduction Parameters							
8	Temperature of O-REx Deck	$T_{O\text{-}REx}$	°C	40	Uniform	-30	50
9	Conductance between O-REx and bracket	G_b	W/m²-C	2,000	Uniform	100	4,000
10	Conductance per screw between bracket and SXM housing	$G_{s,b}$	W/C	0.42	Uniform	0.11	1.32
11	Conductance per screw between pre-amp and SXM housing	$G_{s,pa}$	W/C	0.26	Uniform	0.07	0.80
12	Conductivity of pins on SDD package	k_{pins}	W/m-°C	400	Uniform	350	405
13	Conductance between SDD housing and SXM housing	G_h	W/m²-C	2,000	Uniform	100	4,000
14	Conductance per screw between collimator and SXM housing	$G_{s,coll}$	W/C	0.21	Uniform	0.03	0.42
Radiation Parameters							
15	Solar Flux	ϕ_s	W/m²	1,367	Uniform	700	1,752
16	Collimator Absorptivity	α_c	--	0.50	Uniform	0.31	0.60
17	Collimator Emissivity	ε_c	--	0.80	Uniform	0.78	0.82
18	SDD Housing Absorptivity	α_h	--	0.50	Uniform	0.30	0.52

Fourier Amplitude Sensitivity Testing (FAST)

- Variance-based global sensitivity analysis method
- Can be more efficient to evaluate “main” or “total” effect sensitivity indices over other methods

Explore N-dimensional space of model parameters via search curve defined by parametric equations

s ~ scalar from [-inf,+inf]
G_i ~ transfer function
ω_i ~ frequencies

$$X_i = G_i \sin(w_i s)$$

In classic FAST, main effects sensitivities are approximated via Fourier coefficients

Sensor Importance Study

- **Objective**: identify through analysis which temperature sensors are most important w.r.t. experimental utility
- Procedure can be used to answer:
  - Where to measure?
  - How accurately to measure?
- Value in knowing sensor importance:
  - Sensor could fail during test
  - Addition of redundant sensors for critical locations
  - Testbed may have sensor quantity restrictions
  - Planned sensor may not be possible to install on system

Occurred on SXM – not possible to place RTD on SXM electronics board. Vector of observations, z, is now 5x1.

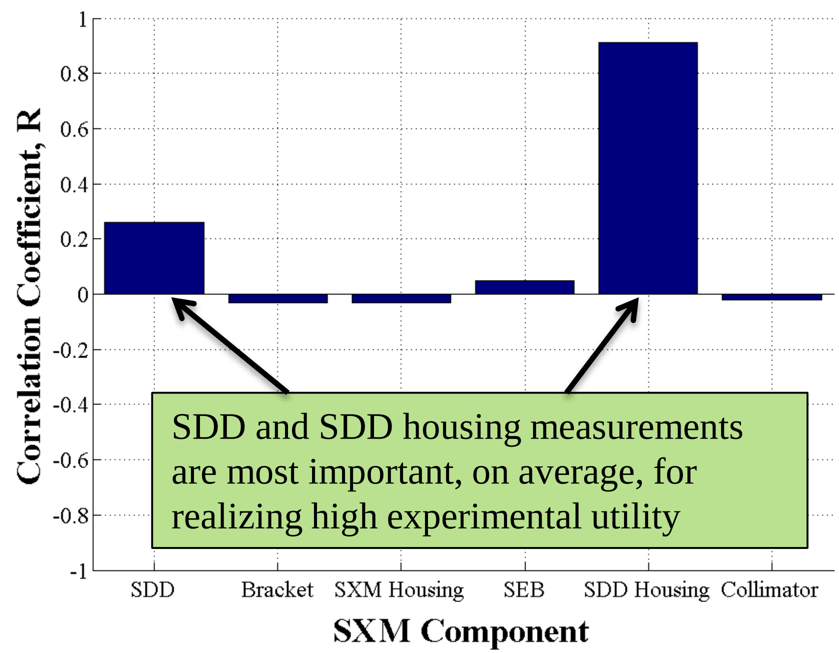
Pearson's Correlation Coefficient

$$R_{A,B} = \frac{cov(A,B)}{\sigma_A \sigma_B}$$

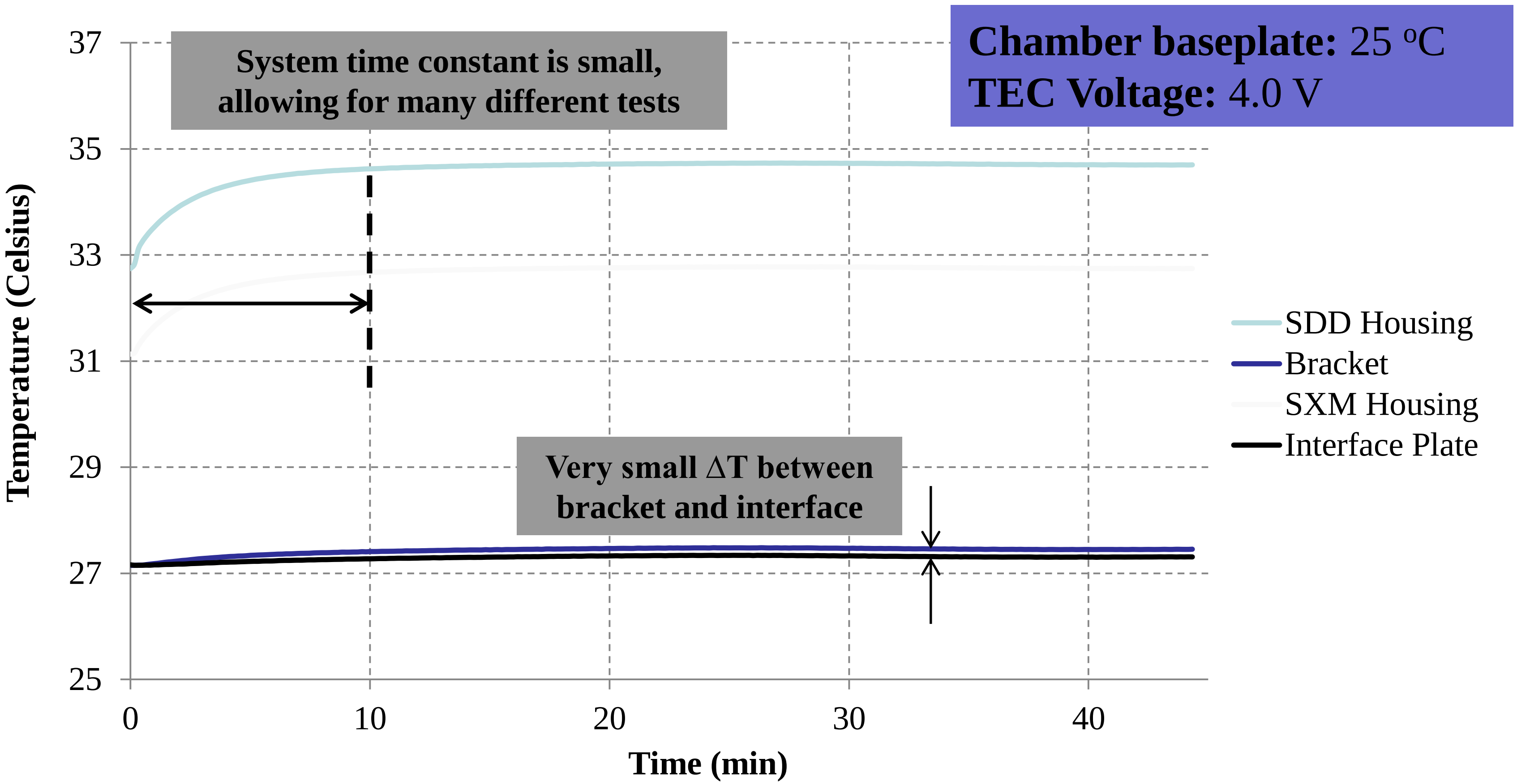
Will indicate whether the presence of a sensor is, on average, correlated to high utility

Matrix of Sensor Permutations (64x6)

$$\begin{array}{cccccc} Bracket & SXMHousing & SEB & SDDHousing & Collimator & SDD \end{array}$$
$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$



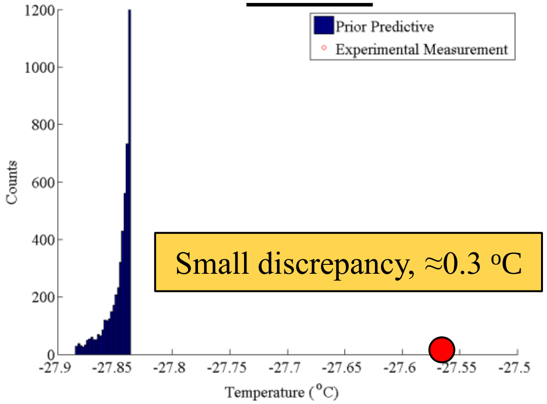
Experimental Results: Sample for T9



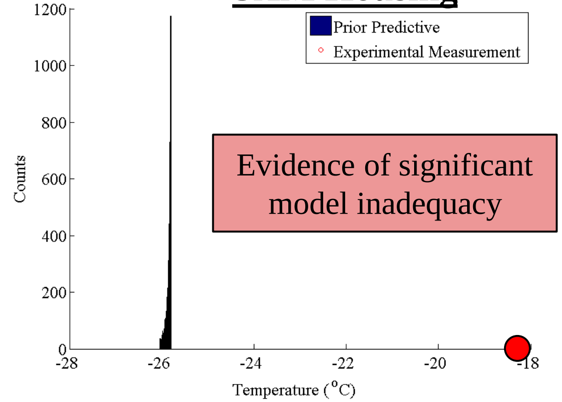
SXM thermal time constant is approximately 10 min due to small thermal capacitance.

Prior Predictive Check (PPC): G_h Only

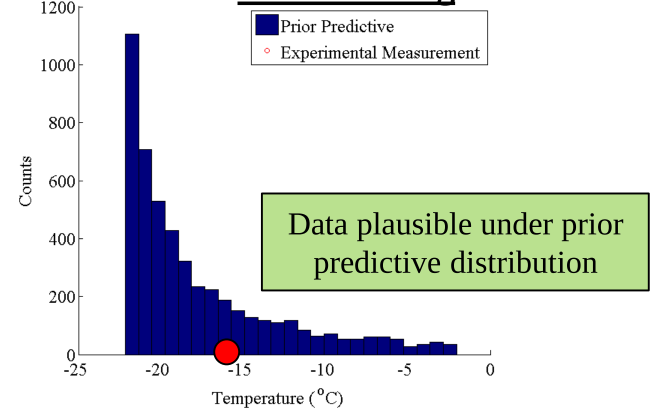
Bracket



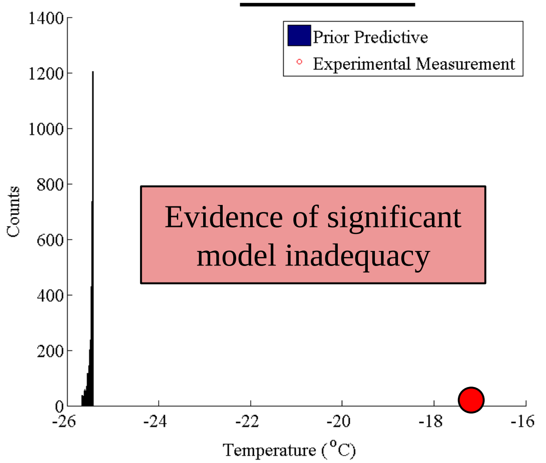
SXM Housing



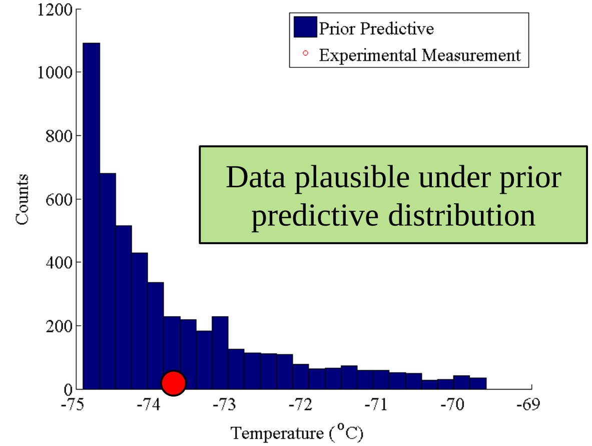
SDD Housing



Collimator



SDD



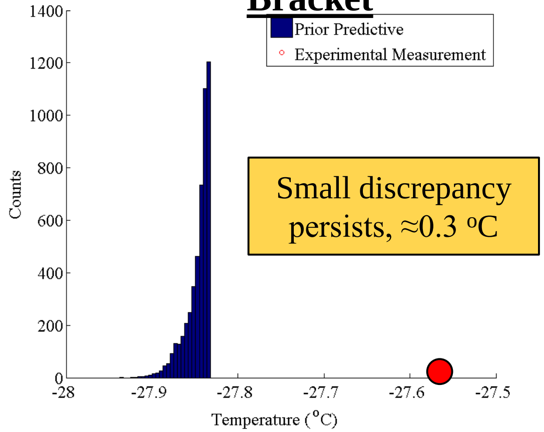
- Propagate prior uncertainty through SXM thermal model
- All parameters have fixed values except for G_h
- PPC for only test phase T36 (4.0 V, -30 ºC)

Current Parametric Model Uncertainty			
Parameter	Units	Min Value	Max Value
G_h	$W/m^2/C$	100	4,000

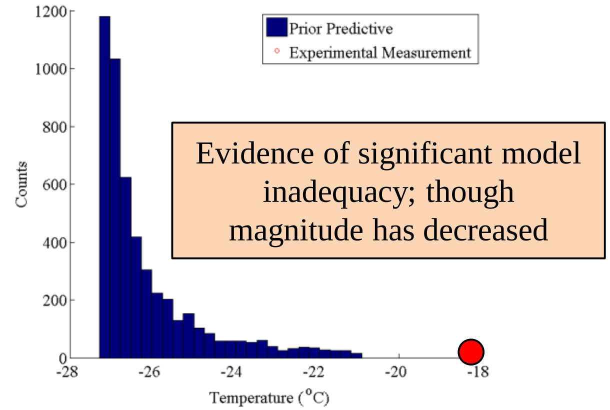
Location of discrepancy and previous GSA suggests to repeat PPC including the uncertainty in conductance between SXM housing and bracket, $G_{s,b}$

PPC: G_h and $G_{s,b}$ Only

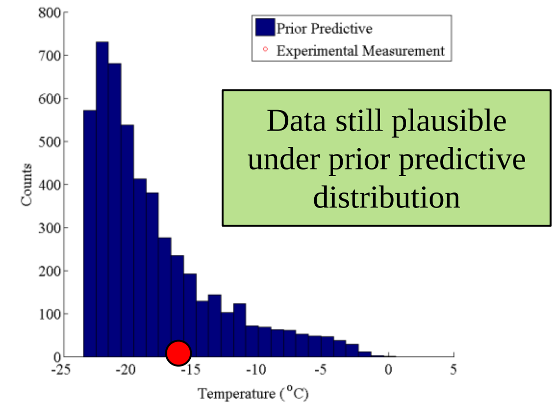
Bracket



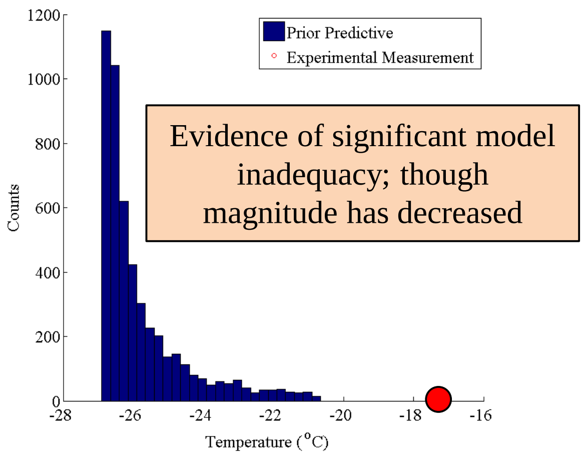
SXM Housing



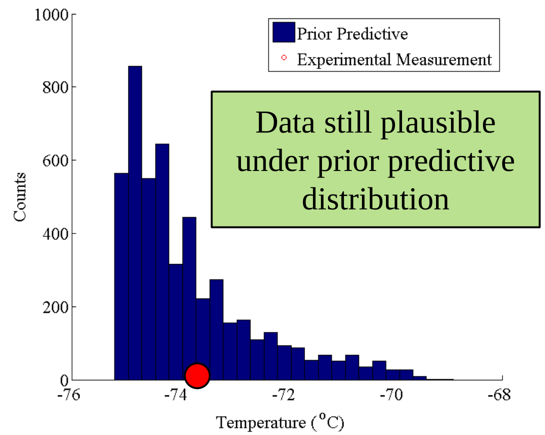
SDD Housing



Collimator



SDD



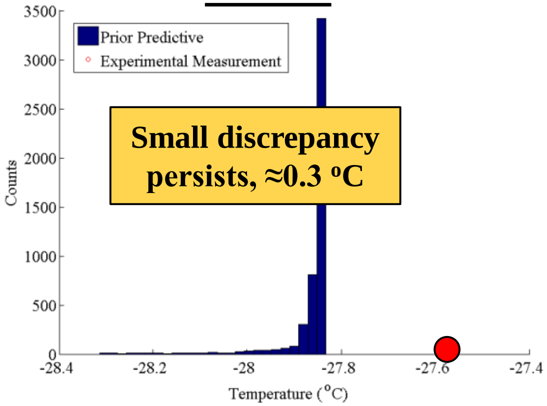
- Propagate prior uncertainty through SXM thermal model
- All parameters have fixed values except for G_h and $G_{s,b}$
- PPC for only test phase T36 (4.0 V, -30 ºC)

Improvement but lack of surface area near screw holes suggests to repeat PPC but reduce lower bound of $G_{s,b}$

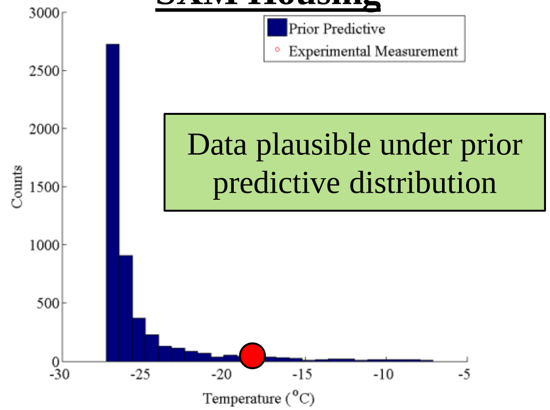
Current Parametric Model Uncertainty			
Parameter	Units	Min Value	Max Value
G_h	$W/m^2/C$	100	4,000
$G_{s,b}$	W/C	0.11	1.32

PPC: G_h and $G_{s,b}$ Only, Relaxed $G_{s,b}$ Lower Bound

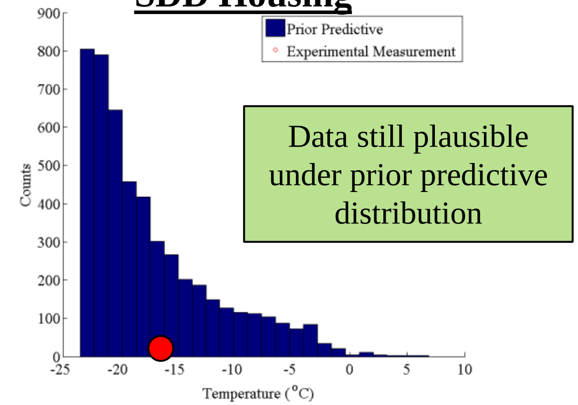
Bracket



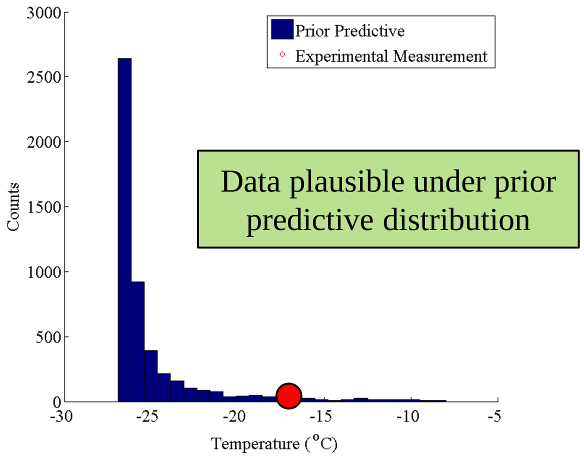
SXM Housing



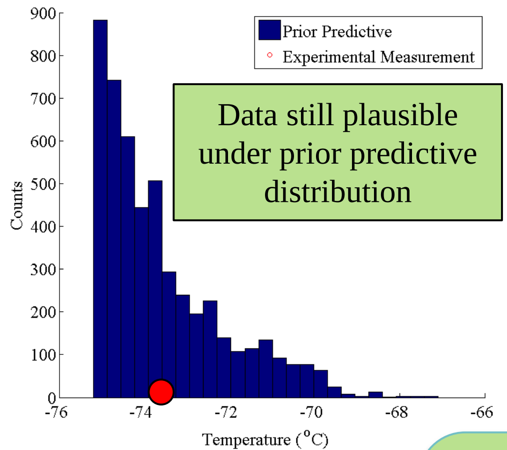
SDD Housing



Collimator



SDD



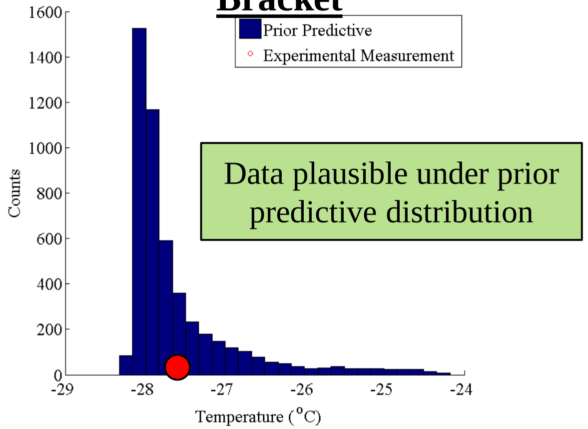
- Propagate prior uncertainty through SXM thermal model
- All parameters have fixed values except for G_h and $G_{s,b}$
- PPC for only test phase T36 (4.0 V, -30 ºC)

Current Parametric Model Uncertainty			
Parameter	Units	Min Value	Max Value
G_h	W/m²/C	100	4,000
$G_{s,b}$	W/C	0	1.32

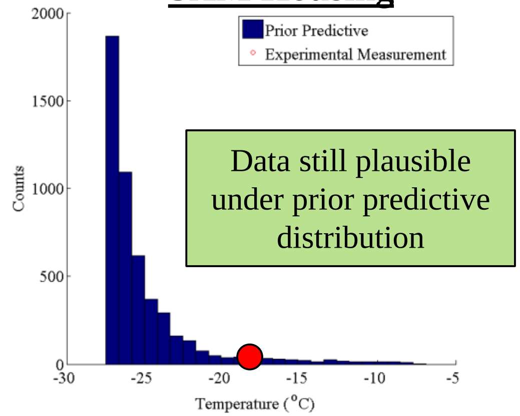
Persisting *small* discrepancy in bracket suggests to repeat PPC including uncertainty in conductance between bracket and interface, G_b

PPC: G_h, $G_{s,b}$ and G_b Only, Relaxed $G_{s,b}$ Lower Bound

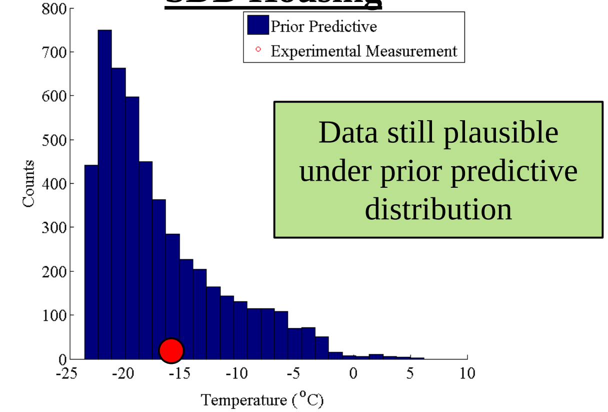
Bracket



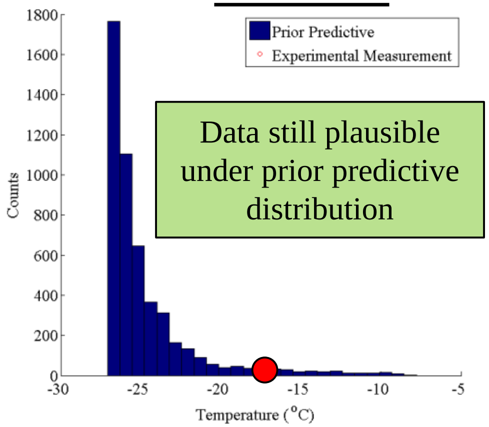
SXM Housing



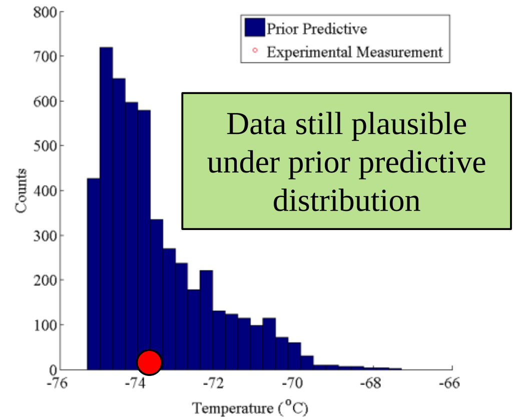
SDD Housing



Collimator



SDD



- Propagate prior uncertainty through SXM thermal model
- All parameters have fixed values except for G_h, $G_{s,b}$ and G_b
- PPC for only test phase T36 (4.0 V, -30 ^oC)

Current Parametric Model Uncertainty			
Parameter	Units	Min Value	Max Value
G_h	$W/m^2/C$	100	4,000
$G_{s,b}$	W/C	0	1.32
G_b	$W/m^2/C$	100	4,000

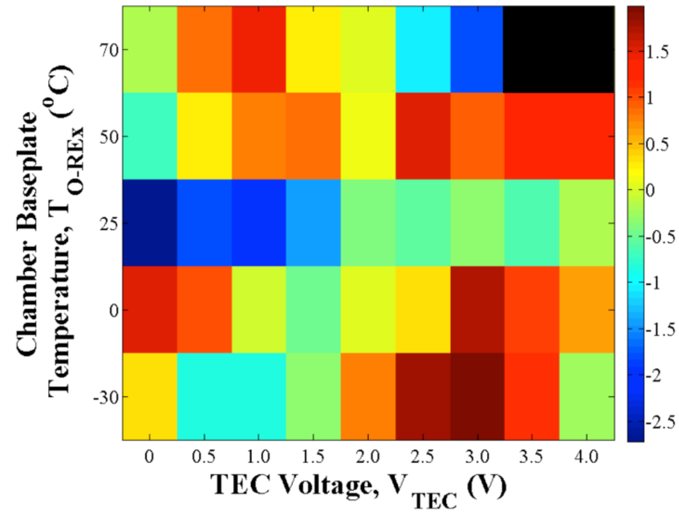
Current parametric uncertainty can explain all data for T36. Now, update parameter distributions and calibrate to *all* test phases.

Quantify Calibrated Model Discrepancy for SDD

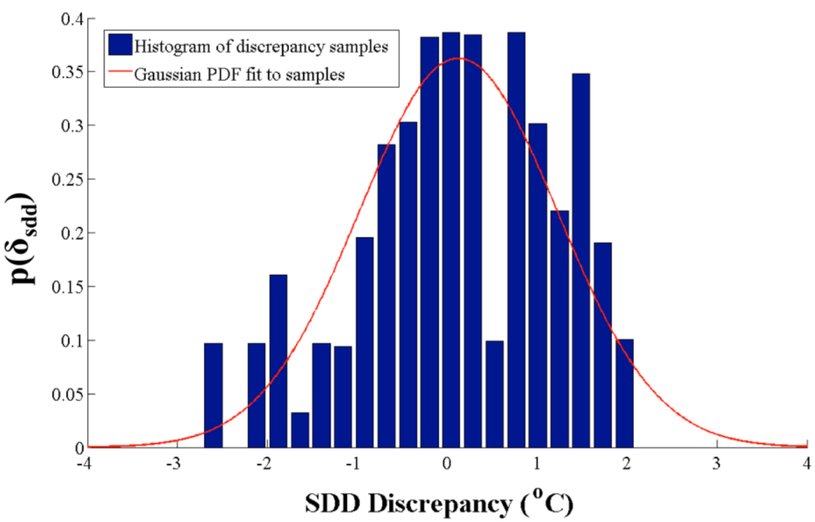
- No obvious functional relationship between SDD temperature and V_{TEC}, $T_{O\text{-}REx}$
- Updated, empirical TEC thermal model under-predicts/over-predicts SDD temperature
  - If more accuracy were required, additional refinements to TEC model would increase predictive accuracy
- Histogram of all discrepancy samples for all 43 test cases reveals that discrepancy can be conservatively captured via Gaussian distribution

SDD discrepancy function will be stationary Gaussian distribution → conservative approach because maximum possible discrepancy variance is considered for all possible V_{TEC}, $T_{O\text{-}REx}$

Mean of SDD Discrepancy Samples for all 43 Test Phases

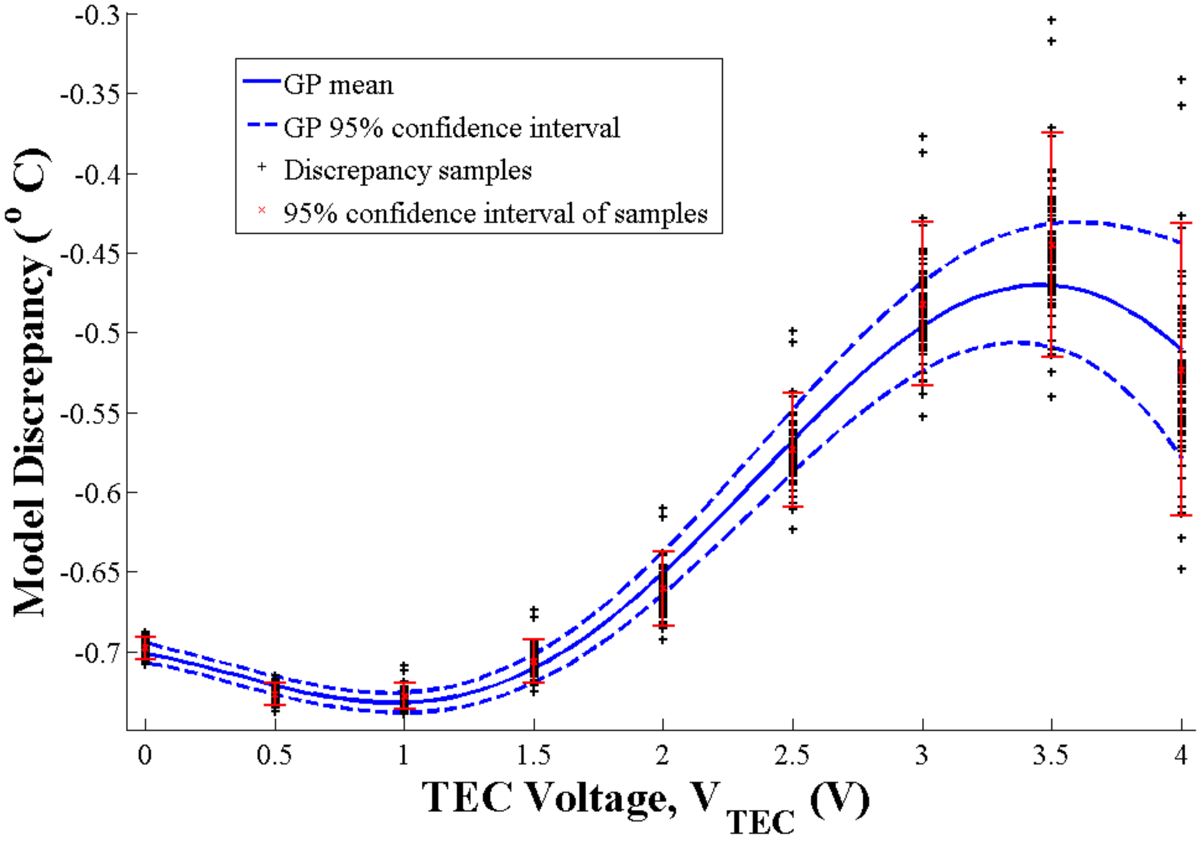


Histogram of SDD Discrepancy Samples for all 43 Test Phases



GP Model – Sample Section

SDD Housing – GP Model Section



Variable voltage and $T_{O\text{-}REx}$ constant at 25 °C

GP model section shows variance reduction and trends in discrepancy samples well-matched with discrepancy model

Comparison of BMV to a Conventional Approach

Validation Step	Analogous BMV Step	BMV	A Conventional Approach
Analysis	**2: UP and Parameter Prioritization**	• All system and environmental parameters probabilistically characterized and propagated through model for many thousands of bounding and intermediate thermal cases; all requirements satisfied for $T_{O\text{-}REx}$ up to 50 ºC • Global sensitivity analysis uses information within model to rigorously, systematically identify critical system sensitivities; SXM conductance G_h is critical sensitivity	• Likely only two analysis cases, corresponding to worst-case hot and cold operational scenarios • Identification of critical system sensitivity up to individual engineer; often manual local sensitivity analysis; heavy reliance on experience/intuition
Test	**4: Design and Implementation of Experiments**	• Parameter inference experiment to maximize information gain in G_h at V_{TEC} = 4.0 V and $T_{O\text{-}REx}$ = -30 ºC • Full factorial model validation experiment with focus on bounding important parameters of domain of intended application of SXM	• System-level thermal balance test at worst hot case, cold case, and possibly a few intermediate cases
Model Update	**5: Experimental Model Calibration and Flight Model Updates**	• SXM thermal model parameters were *updated* (not replaced) via systematic, Bayesian calibration approach • Remaining model inadequacy was quantified via Gaussian Process Models to predict inadequacy for any SXM power mode or spacecraft interface temperature	• Manual correlation or parameter optimization model update procedure • Differences between model predictions and experimental data are less than a threshold value (e.g., ±3 ºC)

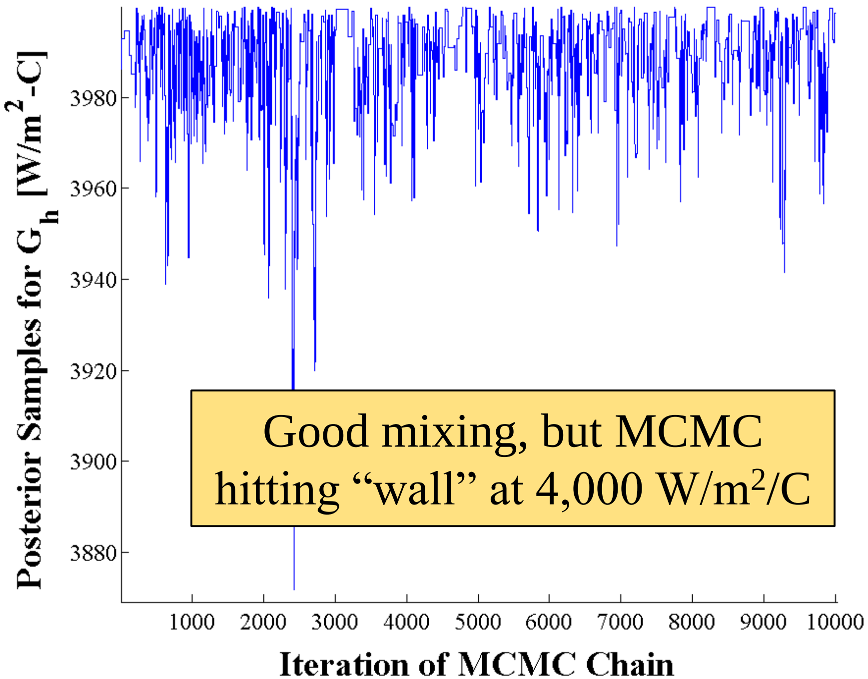
For SXM case study, BMV led to additional information being available to the engineer at each major step of the validation process. BMV focused validation efforts to critical areas of SXM thermal system and provided a more rigorous quantification of model uncertainties before and after testing.

Importance of $T_{O\text{-}REx}$ as System Design Parameter

- Cooling the SXM interface is driving thermal system accommodation for REXIS SXM
  - SXM is nominally facing the sun
  - Need to cool the SXM interface to 50 ^{o}C with the GEVS [17] standard thermal design margin of 5 ^{o}C
- Due to the 50 ^{o}C spacecraft interface upper limit, design changes to OSIRIS-REx included:
  - Heat spreader and RTV added to interface to decrease thermal resistance across interface
  - Changes in surface coatings near the SXM to help cool the mounting structure
  - Redesign of MLI blankets near the interface to increase heat rejection from structure to cooler parts of spacecraft
- Power cycling of REXIS *could* be necessary if temperatures are slightly warmer than expected
  - Operational mission plan has changed since the 50 ^{o}C upper limit was set
  - Power cycling introduces risk to spectrometer detector array that would require major rework to spectrometer electronics so that detectors could remain on if SXM were power cycled

The spacecraft-SXM interface temperature, $T_{O\text{-}REx}$, is an important system design parameter. If the upper limit had been higher, some or all of the design changes and potential operational constraints would not have been necessary.

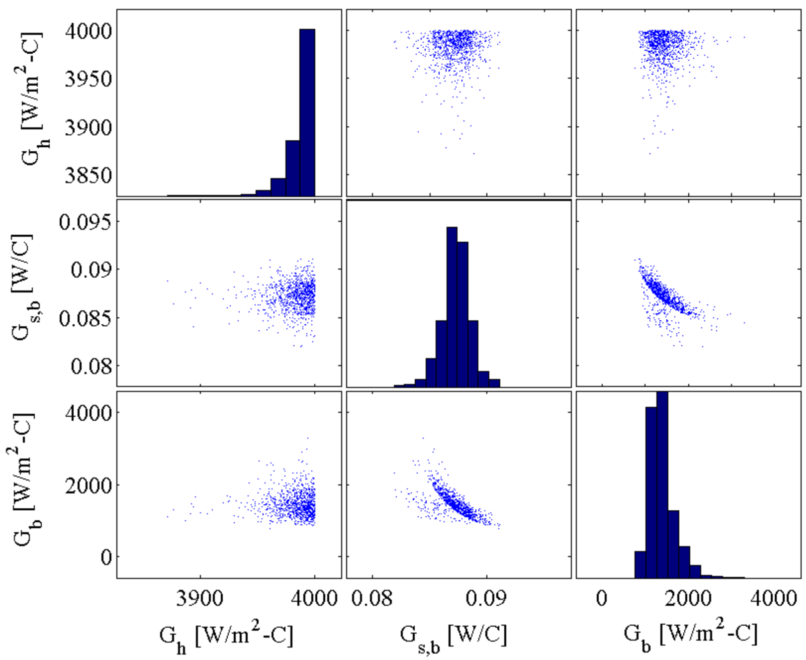
MCMC Results



Good mixing, but MCMC hitting "wall" at 4,000 W/m²/C

Glasgow and Kittredge [30]: Cho-Therm 1671 (applied to G_h interface) tested near its vendor-specified value of 6,700 W/m²/C

Correlated posterior distributions: $G_{s,b}$ and G_b affected by G_h "wall"



Increase upper bound of G_h distribution and update MCMC results